Volumes of Rectangular Prisms

What You'll Learn

To find the volume of rectangular prisms with fractional edge lengths

🔊 **New Vocabulary** volume

Gather your clicker and get a new note in your Evernote math notebook.

Get ready for the warm-up questions.

Find the area of a rectangle with a base of 6 inches and a height of 4 inches.

A) 20 inches squared

B) 10 inches squared

C) 12 inches squared

D) 24 inches squared

Find the area of a triangle with a base of 8 inches and a height of 6 inches.

A 48 inches squared

B 28 inches squared

C 24 inches squared

D 32 inches squared

To find surface area of any polygon, you _____ the area of the sides.

A add

B subtract

C multiply

D divide

E none of the above

Any side can be considered the _____ of a parallelogram

A) height

B) base

C) area

D) segment

Why Learn This?

The volume of a fish tank tells you how much water the tank can hold. The **volume** of a three-dimensional figure is the number of cubic units needed to fill the space inside the figure.
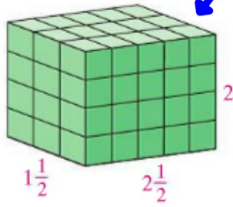
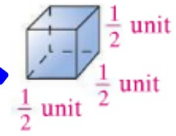**Area is two dimensional (a base and a height), so it is labeled as squared.**

Volume is three dimensional (a length a width and a height) , so it is labeled as cubed.

Counting Cubes to Find Volume

1 Find the volume of the rectangular prism.

Step 1 Choose an appropriate cube. Two of the mixed numbers have fractions with denominators of 2. Use a $\frac{1}{2}$-unit cube.

$\frac{1}{2}$ unit

$\frac{1}{2}$ unit

$\frac{1}{2}$ unit

Step 2 Count the number of cubes in the prism.

$2\frac{1}{2} = \frac{5}{2}$ or $5 \times \frac{1}{2}$ ← There are 5 cubes across the front.

$1\frac{1}{2} = \frac{3}{2}$ or $3 \times \frac{1}{2}$ ← There are 3 cubes along the side.

$2 = \frac{4}{2}$ or $4 \times \frac{1}{2}$ ← There are 4 cubes up.

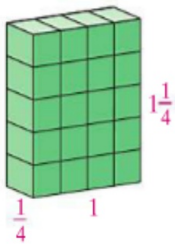So the prism has a total of $5 \times 3 \times 4$, or 60 cubes.

Step 3 Find the volume of one cube.

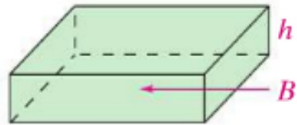The volume of one cube is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{8}$ cubic unit.

Step 4 Multiply the number of cubes by the volume of each cube.

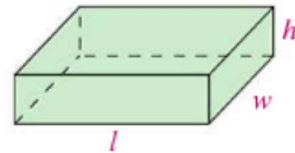$60 \times \frac{1}{8} = \frac{60}{8} = \frac{15}{2} = 7\frac{1}{2}$

The volume of the prism is $7\frac{1}{2}$ cubic units.

$1\frac{1}{2}$ $2\frac{1}{2}$ 2

$1\frac{1}{4}$ $\frac{1}{4}$ 1

You have used formulas to find the volume of prisms whose edges have whole number lengths.

h

B

h

w

l

volume = area of base × height   volume = length × width × height

$V = Bh$ $V = lwh$

You can use the same formulas when the lengths of the prism edges are fractions or mixed numbers.

Find the volume of a rectangular prism whose length is 5 meters, width is 2 meters, and height is 4 meters.

Find the volume of a cube whose sides are 4 inches each.

A) 12 inches cubed

D) 48 inches cubed

B) 24 inches cubed

E) 64 inches cubed

C) 16 inches cubes

F) 32 inches cubed

Think about a cube whose sides each measure 3 inches. The volume could be found by multiplying 3 times 3 times 3. The product would be 27 inches cubed.

Talk at your table about what would happen to the volume of that cube if all of its sides were doubled.

Finally, talk at your table about what would happen to the volume of that cube it all of its sides were tripled.

$1\frac{1}{3}$ ft

2 ft

$2\frac{2}{3}$ ft

EXAMPLE   **Finding the Volume of a Prism**

2 **Fish Tanks** Find the volume of the fish tank shown at the left.

Method 1 Count the cubes.

Choose an appropriate sized cube. Two of the mixed numbers have fractions with denominators of 3. Use a $\frac{1}{3}$-foot cube.

$2\frac{2}{3} = \frac{8}{3}$ ← There are 8 cubes across the front.

$1\frac{1}{3} = \frac{4}{3}$ ← There are 4 cubes along the side.

$2 = \frac{6}{3}$ ← There are 6 cubes up.

So the prism has a total of $8 \times 4 \times 6$, or 192 cubes.

The volume of a $\frac{1}{3}$-foot cube is $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$, or $\frac{1}{27}$ cubic foot.

$192 \times \frac{1}{27} = \frac{192}{27} = \frac{64}{9} = 7\frac{1}{9}$

The volume of the prism is $7\frac{1}{9}$ cubic feet.

Method 2 Use a formula.

$V = l \times w \times h$ ← Use the formula for the volume of a rectangular prism.

$= 2\frac{2}{3} \times 1\frac{1}{3} \times 2$ ← Substitute $2\frac{2}{3}$ for l, $1\frac{1}{3}$ for w, and 2 for h.

$= \frac{8}{3} \times \frac{4}{3} \times \frac{2}{1}$ ← Multiply.

$= \frac{64}{9} = 7\frac{1}{9}$

The volume is $7\frac{1}{9}$ cubic feet, or $7\frac{1}{9}$ ft³.

2. Find the volume of a rectangular prism with a length of $8\frac{1}{5}$ meters, a width of 7 meters, and a height of $10\frac{2}{5}$ meters.

✓ **Check Your Understanding**

1. **Vocabulary** How are volume and area different?

2. **Number Sense** How does the volume of a cube change if its dimensions are doubled?

3. What is the volume of a $\frac{1}{5}$-cm cube?

Find the volume of a rectangular prism whose length is 8 1/2 meters, width is 4 meters and height is 3 1/2 meters.

A 119 meters cubed

D 109 meters cubed

B 96 1/2 meters cubed

E 104 1/2 meters cubed

C 97 meters cubed

F 109 1/4 meters cubed

Power down your clickers and put them away.

I will pass out a guided problem solving practice sheet for us to work on to increase our skills. We will complete as much of it together as time allows. What we do not finish in class will become homework, due tomorrow.

8-5 • Guided Problem Solving

GPS Student Page 302, Exercise 22:

A truck trailer has a length of 20 feet, a width of $8\frac{1}{2}$ feet, and a height of $7\frac{1}{2}$ feet. A second trailer has a base area of 108 square feet and a height of $8\frac{1}{2}$ feet. Which trailer has a greater volume? How much greater is it?

Understand

1. Circle the information you will need to solve the problem.

2. Write the formula used to find the volume of a rectangular solid.

   _____

Plan and Carry Out

3. Substitute the values for the length, width, and height of the first trailer into the formula for the volume of a rectangular solid. What is the volume?

   _____

4. Repeat Step 3 for the second trailer.

   _____

5. What are the units for the volume of this solid?

   _____

6. Which trailer has the greater volume?

   _____

7. How much greater is the volume?

   _____

Check

8. How can you check your answer?

   _____
   _____

Solve Another Problem

9. A building is $32\frac{1}{2}$ feet tall and has a base area of 420 square feet. What is the volume of the building?

   _____