Algebra

2-5 Types of Solutions of Linear Equations

CONTENT STANDARDS
8.EE.7, 8.EE.7.a, 8.EE.7.b

What You'll Learn

To identify whether a linear equation in one variable has one, infinitely many, or no solutions

Why Learn This?

Equations with variables on both sides can help you check whether two ways of paying for something, such as swimming lessons, cost the same.

The equations solved in this chapter so far have resulted in a variable equal to a number.
Sometimes, solving an equation may result in a number equal to the same number, or a different number. Each of these results indicates how many solutions an equation has. This is summarized in the table below, where x represents the variable and a and b represent different numbers.

Copy this table into your notes:

KEY CONCEPTS Types of Solutions

Algebraic Form	Number of Solutions	Description
$a = b$	None	There are *no* values of the variable for which the equation is true.
$x = a$	One	The equation is true for *exactly one* value of the variable.
$a = a$	Infinitely many	The equation is true for *all* values of the variable.

EXAMPLE 1 Identifying Types of Solutions

Tell whether each equation has one solution, infinitely many solutions, or no solution. Justify your answer.

a.

$$
\begin{aligned}
2x - 4 &= -x - 1 \\
2x + 1x - 4 &= -x + 1x - 1 && \leftarrow \textbf{Add 1}\boldsymbol{x}\textbf{ to each side.} \\
3x - 4 &= -1 && \leftarrow \textbf{Simplify.} \\
3x - 4 + 4 &= -1 + 4 && \leftarrow \textbf{Add 4 to each side.} \\
3x &= 3 && \leftarrow \textbf{Simplify.} \\
x &= 1 && \leftarrow \textbf{Divide both sides by 3.}
\end{aligned}
$$

The result is an equation of the form $x = a$. This equation is true for exactly one value. So, the equation has one solution.

b.

$$
\begin{aligned}
2x - 4 &= 2(x - 2) \\
2x - 4 &= 2x - 4 && \leftarrow \textbf{Use the Distributive Property.} \\
2x - 4 - 2x &= 2x - 4 - 2x && \leftarrow \textbf{Subtract } 2x \textbf{ from each side.} \\
-4 &= -4 && \leftarrow \textbf{Simplify.}
\end{aligned}
$$

The result is an equation of the form $a = a$. This equation is true for all values of x. So, the equation has infinitely many solutions.

c.

$$2x - 4 = 2(x + 1)$$

$$2x - 4 = 2x + 2 \quad \leftarrow \text{ Use the Distributive Property.}$$

$$2x - 4 - 2x = 2x + 2 - 2x \quad \leftarrow \text{ Subtract } 2x \text{ from each side.}$$

$$-4 = 2 \quad \leftarrow \text{ Simplify.}$$

The result is an equation of the form $a = b$. There are no values of x for which the equation is true. So, the equation has no solution.

1 Identifying Types of Solutions Tell whether each equation has one solution, infinitely many solutions, or no solution. Justify your answer.

a.

$$6w + 3 = 4w - 1$$

$$6w - \boxed{4w} + 3 = 4w - \boxed{4w} - 1$$ ← **Subtract 4*w* from each side.**

$$\boxed{2w} + 3 = -1$$ ← **Simplify.**

$$2w + 3 - 3 = -1 - \boxed{3}$$ ← **Subtract 3 from each side.**

$$2w = -4$$ ← **Simplify.**

$$\frac{2w}{2} = \frac{-4}{2}$$ ← **Divide each side by $\boxed{2}$.**

$$w = \boxed{-2}$$ ← **Simplify.**

The result is an equation of the form $x = a$. So, the equation has $\boxed{\text{one}}$ solution(s).

b.

$$6w + 3 = 6(w + 0.5)$$

$$6w + 3 = 6w + \boxed{3} \quad \leftarrow \text{Use the Distributive Property.}$$

$$6w - \boxed{6w} + 3 = 6w - \boxed{6w} + 3 \quad \leftarrow \text{Subtract } 6w \text{ from each side.}$$

$$3 = \boxed{3} \quad \leftarrow \text{Simplify.}$$

The result is an equation of the form $a = a$. So, the equation has [infinitely many] solution(s).

c.

$$6w + 3 = 6(w - 4)$$

$$6w + 3 = \boxed{6w} - 24 \quad \leftarrow \textbf{Use the Distributive Property.}$$

$$6w - 6w + 3 = 6w - \boxed{6w} - 24 \quad \leftarrow \textbf{Subtract } \boxed{6w} \textbf{ from each side.}$$

$$3 = \boxed{-24} \quad \leftarrow \textbf{Simplify.}$$

The result is an equation of the form $a = b$. So, the equation has [no] solution(s).

Quick Check

1. Tell whether each equation has one solution, infinitely many solutions, or no solution. Justify your answer.

a. $5x + 8 = 5(x + 3)$ **b.** $9x = 8 + 5x$ **c.** $6x + 12 = 6(x + 2)$ **d.** $7x - 11 = 11 - 7x$

EXAMPLE 2 Application: Comparing Costs

Sports You want to take 10 lessons at a swim club. You can pay a membership fee of \$20 plus a fee per lesson. You can also decide not to pay a membership fee. In that case, the fee per lesson is \$3 more. Is there any lesson fee for which these two plans cost the same? Justify your answer.

Words membership fee + 10 lessons · lesson fee = 10 lessons · (lesson fee + \$3)

Equation Let f = lesson fee.

$$20 + 10 \cdot f = 10 \cdot (f + 3)$$

$$20 + 10f = 10(f + 3)$$

$$20 + 10f = 10f + 30 \quad \leftarrow \text{Use the Distributive Property.}$$

$$20 + 10f - 10f = 10f + 30 - 10f \quad \leftarrow \text{Subtract } 10f \text{ from each side.}$$

$$20 = 30 \quad \leftarrow \text{Simplify.}$$

The result is an equation of the form $a = b$. So the equation has no solution. There is no lesson fee for which these two plans cost the same.

Example

2 **Comparing Costs** A movie club card costs \$10. Cardholders pay the member's cost for a movie ticket. A person without a club card pays \$2 more for a ticket. A friend tells you that the cost of 5 tickets is the same for both members and nonmembers. Is this true? Justify your answer.

Words cost of club card + 5 · member's ticket price = 5 · (member's ticket price + \$2)

Let t = cost of one member's ticket in dollars.

Equation

$\boxed{10} + 5t = 5(t + 2)$ ← **Substitute 10 for cost of card in dollars.**

$10 + 5t = \boxed{5t} + 10$ ← **Use the Distributive Property.**

$10 + 5t - \boxed{5t} = 5t - \boxed{5t} + 10$ ← **Subtract 5*t* from each side.**

$10 = 10$ ← **Simplify.**

The result is an equation of the form $\boxed{a} = \boxed{a}$. So, the equation has infinitely many solution(s), and the statement is $\boxed{\text{true}}$.

Talk with your table partners to solve these:

Match each equation to the correct number of solutions.

2. $4x + 8 = 4(x + 4)$

3. $5x = 9 + 2x$

4. $x + 9 = 7x + 9 - 6x$

A. one

B. infinitely many

C. none

7x = 3x - 12

A. One solution

B. Infinitely many solutions

C. No solution

$3x + 3 = 3(x + 1)$

A. One solution

B. Infinitely many solutions

C. No solution

22y = 11(3 + y)

A. One solution

B. Infinitely many solutions

C. No solution

$-3t + 1 = t + 9 - 4t$

A) One solution

B) Infinitely many solutions

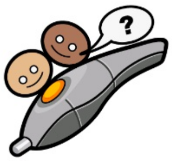
C) No solution

$4(z - 3) = 2(2z - 5)$

A. One solution

B. Infinitely many solutions

C. No solution



$1.8k - 2 = 0.2(3k - 4)$

- A. One solution
- B. Infintely many solutions
- C. No solution

Power down your clickers and put them away.

No assignment today.