8-4 Transformations and Congruence

What You'll Learn

To describe a sequence of transformations that maps one figure onto another; to determine whether two figures are congruent by using a sequence of transformations

CONTENT STANDARDS
8.G.2

Why Learn This?

When you walk in the sand, you leave a trail of footprints that are congruent to each other. You can use transformations to map one footprint onto another.

If two figures are congruent, then a transformation, or a sequence of transformations, will map one figure onto the other.

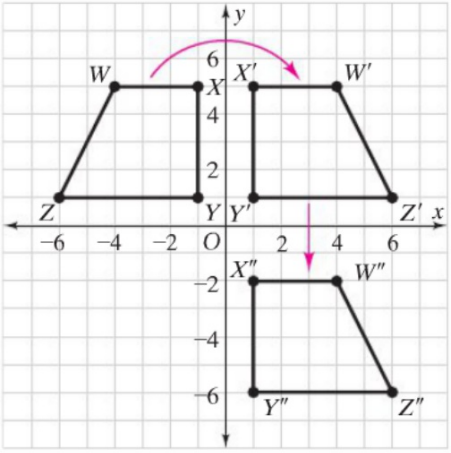
EXAMPLE 1 Recognizing a Series of Transformations

The three trapezoids are congruent. Describe the sequence of transformations that maps $WXYZ$ onto $W''X''Y''Z''$.

A reflection over the y-axis maps $WXYZ$ onto $W'X'Y'Z'$.

A translation 7 units down maps $W'X'Y'Z'$ onto $W''X''Y''Z''$.

So, a reflection over the y-axis, followed by a translation 7 units down, maps $WXYZ$ onto $W''X''Y''Z''$.



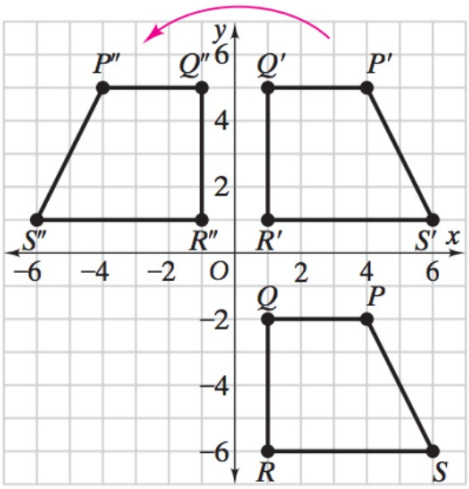
Example

1 Recognizing a Series of Transformations The three trapezoids are congruent. Describe the sequence of transformations that maps $PQRS$ onto $P'''Q'''R'''S'''$.

A translation [7] units [up] maps $PQRS$ onto [$P'Q'R'S'$].

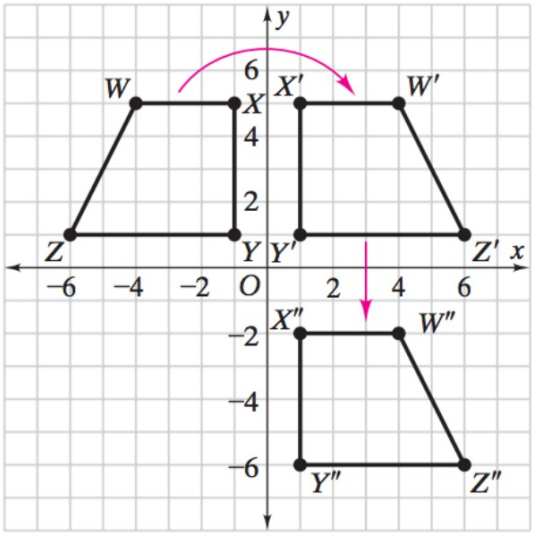
A reflection over the [y]-axis maps $P'Q'R'S'$ onto [$P''Q''R''S''$].

So, a translation [7] units [up], followed by a [reflection] over the [y]-axis, maps $PQRS$ onto [$P''Q''R''S''$].



Quick Check

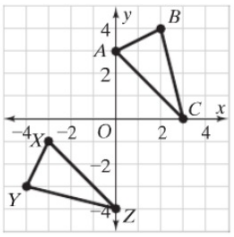
1. Describe the sequence of transformations that maps $WXYZ$ onto $W''X''Y''Z''$.



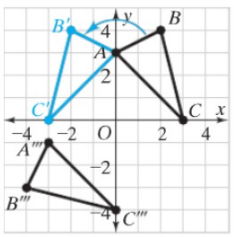
If you can use a sequence of transformations to map one figure onto another, then the two figures are congruent.

EXAMPLE Using Transformations to Determine Congruence

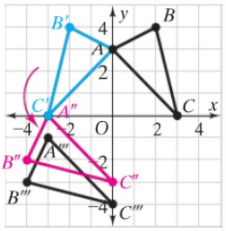
Determine whether the two triangles in the diagram are congruent. If they are, write a congruence statement. If they are not congruent, explain why.



$\triangle ABC$ and $\triangle XYZ$ have opposite orientations and are on opposite sides of the y-axis, so start by reflecting $\triangle ABC$ over the y-axis to get $\triangle A'B'C'$.

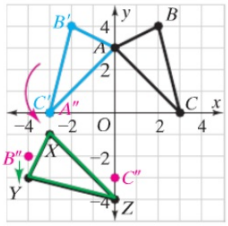


Since $\triangle A'B'C'$ and $\triangle XYZ$ are in different positions, rotate $\triangle A'B'C'$ 90° about the origin to get $\triangle A''B''C''$.



Each vertex of $\triangle XYZ$ is 1 unit down from the corresponding vertex of $\triangle A''B''C''$. So translating $\triangle A''B''C''$ 1 unit down will map it onto $\triangle XYZ$.

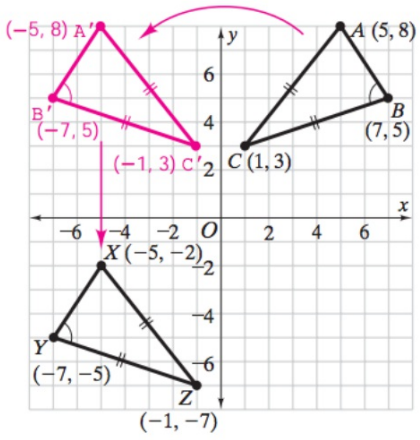
A reflection over the y-axis, followed by a rotation of 90° about the origin, followed by a translation 1 unit down maps $\triangle ABC$ onto $\triangle XYZ$. So, $\triangle ABC \cong \triangle XYZ$.



Example

❷ Using Transformations to Determine Congruence

Determine whether the two triangles in the diagram are congruent. If they are congruent, write a congruence statement. If they are not congruent, explain why.



$\triangle ABC$ and $\triangle XYZ$ have [opposite] orientations and are on [opposite] sides of the y-axis.

So, first [reflect] $\triangle ABC$ over the [y]-axis to get $\triangle$[$A'B'C'$].

Then, [translate] the reflected image $\triangle$[$A'B'C'$] [10] units [down] to map it onto $\triangle$[XYZ].

Finally, write your congruence statement:

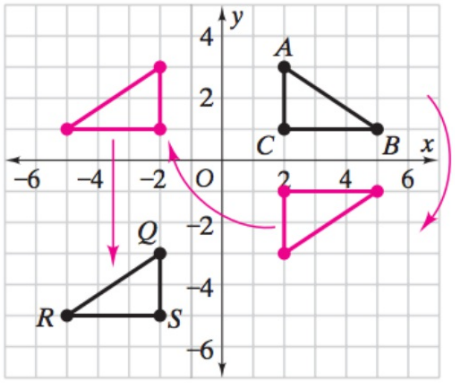
A reflection over the y-axis followed by a translation 10 unit down maps $\triangle ABC$ onto $\triangle XYZ$.

Quick Check

2. Determine whether $\triangle ABC$ is congruent to $\triangle QRS$. If the triangles are congruent, tell what sequence of transformations will map $\triangle ABC$ onto $\triangle QRS$. Then write a congruence statement. If they are not congruent, explain why.

A reflection over the *x*-axis, followed by a rotation of 180° about the origin, followed by a translation of 6 units down maps $\triangle ABC$ onto $\triangle QRS$.

$\triangle ABC \cong \triangle QRS$.



You are to log into your Manga High account and play Transtar to practice these skills.