8-5 Surface Areas of Pyramids and Cones

What You'll Learn

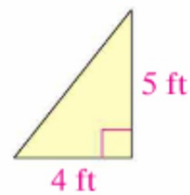
To find surface areas of pyramids and cones using nets and formulas

New Vocabulary slant height

You will need clickers, Evernote, and your Desmos Calculator App open for today's lesson.

Check Skills You'll Need

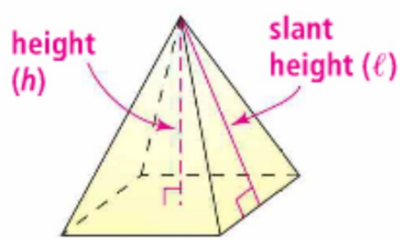
1. **Vocabulary Review** The longest side of a right triangle is the __?__.
2. Find the area of the figure below to the nearest whole unit.



Why Learn This?

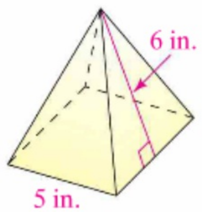
When you can find the surface area of pyramids and cones, you can find the amount of materials you need for projects as large as roofing a house or as small as making a funnel.

The height of a pyramid is different from the height of its lateral faces. For this reason, the height of a pyramid's lateral faces is called the **slant height** and is indicated by the symbol ℓ.



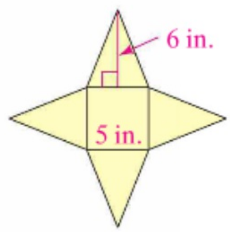
You can draw a net to find the surface area of a square pyramid. The four triangular faces are congruent isosceles triangles.

EXAMPLE Using a Net to Find Surface Area



1 Find the surface area of the square pyramid at the left.

Step 1 Draw a net of the pyramid.



Step 2 Find the area of the faces and the base.

S.A. = area of triangles + area of square

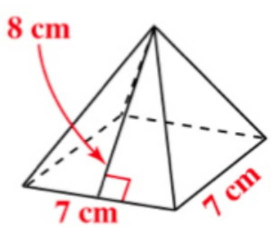
$= 4 \cdot \frac{1}{2}bh + s^2$

$= 4 \cdot \frac{1}{2}(5 \cdot 6) + 5^2$ ← **Substitute 5 for *b*, 6 for *h*, and 5 for *s*.**

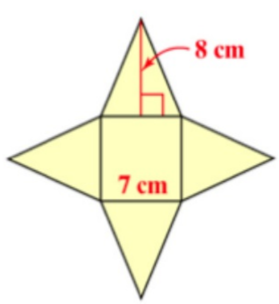
$= 60 + 25$ ← **Simplify.**

$= 85$ ← **Add.**

1 EXAMPLE Find the surface area of this square pyramid.



First draw a net of the pyramid.



Then find the area of the faces and the base.

S.A. = 4 • area of triangle + area of square

$= 4 \cdot \frac{1}{2}bh + s^2$

$= 4 \cdot \frac{1}{2}(7 \cdot 8) + 7^2$ ← Substitute.

$= 112 + 49$ ← Simplify.

S.A. = 161 ← Add.

The surface area is 161 cm^2.

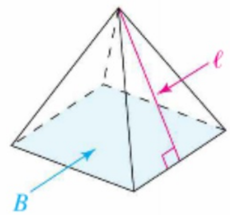
Put the key concepts into your Evernote notes.

You can also use a formula to find the surface area of a square pyramid.

KEY CONCEPTS **Lateral Area and Surface Area of a Square Pyramid**

The lateral area L.A. of a square pyramid is four times the area of one of the lateral faces.

$$\text{L.A.} = 4 \cdot \left(\tfrac{1}{2}b\ell\right) = 2b\ell$$



The surface area S.A. of a square pyramid is the sum of the lateral area and the area of the base.

$$\text{S.A.} = \text{L.A.} + B$$

EXAMPLES **Finding Lateral and Surface Area**

2 **Architecture** The photo at the right shows the Pyramid Arena in Tennessee. Find the lateral area to determine the amount of siding material it needs.

$\text{L.A.} = 2b\ell$ ← lateral area formula

$= 2(450)(367)$ ← Substitute 450 for b and 367 for ℓ.

$= 330{,}300$ ← Simplify.

The lateral area of the Pyramid Arena is 330,300 ft^2.

3 Find the surface area of the Pyramid Arena.

$\text{S.A.} = \text{L.A.} + B$ ← surface area formula

$= 2b\ell + b^2$ ← Use $2b\ell$ for L.A. and b^2 for B.

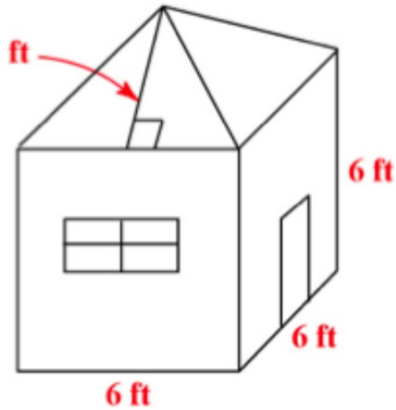
$= 2(450)(367) + 450^2$ ← Substitute 450 for b and 367 for ℓ.

$= 330{,}300 + 202{,}500$ ← Simplify.

$= 532{,}800$ ← Add.

The surface area is 532,800 ft^2.

2 EXAMPLE Find the lateral area of the roof of the playhouse to determine the amount of roofing material needed.



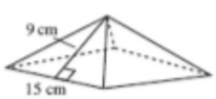
$L.A. = 2b\ell$ ← lateral area formula

$= 2(6)(3.5)$ ← Substitute.

$= 42$ ← Multiply.

The lateral area of the roof of the playhouse is 42 ft^2.

3 EXAMPLE Find the surface area of the square pyramid.



$S.A. = L.A. + 2B$ ← surface area formula

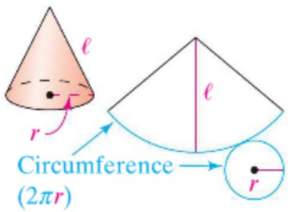
$= 2b\ell + b^2$ ← Use $2b\ell$ for L.A. and b^2 for B.

$= 2(15)(9) + 15^2$ ← Substitute 15 for b and 9 for ℓ.

$= 270 + 225$ ← Use the order of operations.

$= 495$ ← Simplify.

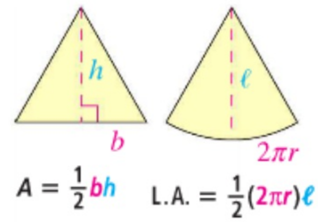
The surface area of the square pyramid is 495 cm^2.



The curved surface of a cone is its lateral surface. In the net at the left, the cone's lateral surface may remind you of a triangle.

The height of the lateral surface is the slant height ℓ. The length of the base of the surface is the circumference of the circular base, $2\pi r$.

You can substitute ℓ and $2\pi r$ in the formula for area of a triangle to find the lateral area of a cone.



$$\text{L.A.} = \frac{1}{2}bh \qquad \leftarrow \text{area of a triangle}$$

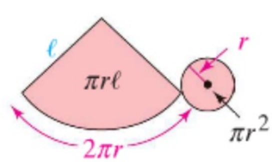
$$= \frac{1}{2}(2\pi r)\ell \qquad \leftarrow \text{Substitute } 2\pi r \text{ for } b \text{ and } \ell \text{ for } h.$$

$$= \pi r\ell \qquad \leftarrow \text{Simplify.}$$

Put the key concepts into your Evernote notes.

KEY CONCEPTS **Lateral Area and Surface Area of a Cone**

The lateral area L.A. of a cone is one half the product of the circumference of the base and the slant height.



$$\text{L.A.} = \frac{1}{2}(2\pi r)\ell = \pi r\ell$$

The surface area S.A. of a cone is the sum of the lateral area and the area of the base.

$$\text{S.A.} = \text{L.A.} + B$$

EXAMPLE Using the Cone Surface Area Formula

4 Find the surface area of the cone at the right to the nearest square meter.

$\text{S.A.} = \text{L.A.} + B$ ← surface area formula

$= \pi r\ell + \pi r^2$ ← Use $\pi r\ell$ for L.A. and πr^2 for B.

$= \pi(7)(30) + \pi(7^2)$ ← Substitute 7 for r and 30 for ℓ.

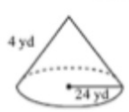
$= 210\pi + 49\pi$ ← Use the order of operations.

$= 259\pi$ ← Simplify.

≈ 813.6724973 ← Use a calculator.

The surface area of the cone is about 814 m^2.

4 EXAMPLE Find the surface area of the cone to the nearest whole unit.



$\text{S.A.} = \text{L.A} + 2B$ ← surface area formula

$= \pi r\ell + \pi r^2$ ← Use $\pi r\ell$ for L.A. and πr^2 for B.

$= \pi(24)(4) + \pi(24)^2$ ← Substitute 24 for r and 4 for ℓ.

$= 96\pi + 576\pi$ ← Use the order of operations.

$= 672\pi$ ← Simplify.

≈ 2111.150263 ← Use a calculator.

The surface area of the cone is about 2,111 yd^2.

Turn on your clickers - get your notes up alongside your desmos caculator app.

1. Which of the following best describes the difference between the lateral area of a square pyramid and the surface area of a square pyramid?

   A. lateral surface area is more than surface area because it does include the area of the square base

   B. lateral surface area is less than surface area because it does not include the area of the square base

   C. lateral surface area is always equal to surface area

   D. lateral surface area is sometimes greater than and sometimes less than surface area

2. A square pyramid has a lateral area of 10.25 m^2 and a base area B of 5.3 m^2. What is its surface area?

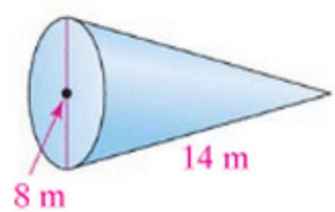
A. 54.325 m^2

B. 31.1 m^2

C. 15 m^2

D. 15.55 m^2

3. What is the slant height of the cone in the accompanying figure?



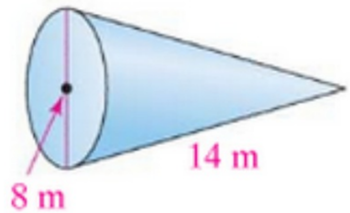
A. 8 m

B. 28 m

C. 22 m

D. 14 m

4. Which of the following best describes the *lateral surface area* of the cone in the figure?



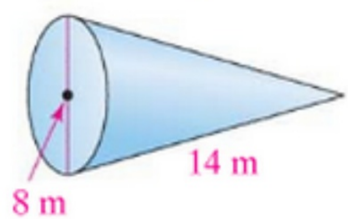
A. $L.A. = (4\pi)(28) = 112\pi$

B. $L.A. = (16\pi)(14) = 224\pi$

C. $L.A. = (4\pi)(14) = 56\pi$

D. $L.A. = (8\pi)(14) = 112\pi$

5. Which of the following best describes the *surface area* of the cone in the figure?



A. $S.A. = 56\pi + 16\pi = 72\pi$

B. $S.A. = 112\pi + 8\pi = 120\pi$

C. $S.A. = 56\pi + 8\pi = 64\pi$

D. $S.A. = 112\pi + 16\pi = 128\pi$

Powerdown your clickers and put them away.

No assignment.