5-3 Solving Systems by Elimination

CONTENT STANDARDS
8.EE.8.b, 8.EE.8.c

What You'll Learn

To solve a system of linear equations by elimination

New Vocabulary elimination method

You will be taking notes on three ways to solve systems by eliminate:

1. *adding to zero out one of the co-efficients*
2. *subtracting to zero out one of the co-efficients, and*
3. *multiplying by a factor to make co-efficients that can be zeroed out by either addition or subtraction*

Why Learn This?

You can use a system of equations to model the amount two students spend on school supplies. One way to solve a system is by using the Addition and Subtraction Properties of Equality to eliminate one of the variables. This is called the **elimination method**.

EXAMPLE 1 Solving a System by Adding

Solve the system by elimination. $5x - 6y = -32$, $3x + 6y = 48$

Step 1 Eliminate one variable. Since the coefficients of y are additive inverses, add to eliminate y.

$$\begin{array}{rrcll} & 5x - 6y & = & -32 & \\ + & 3x + 6y & = & 48 & \\ \hline & 8x + 0 & = & 16 & \leftarrow \textbf{Add.} \\ & x & = & 2 & \leftarrow \textbf{Solve for } x. \end{array}$$

Step 2 Substitute 2 for x in either equation and solve for y.

$$\begin{array}{rcll} 5x - 6y & = & -32 & \leftarrow \textbf{Write either equation.} \\ 5(2) - 6y & = & -32 & \leftarrow \textbf{Substitute 2 for } x. \\ 10 - 6y & = & -32 & \leftarrow \textbf{Simplify.} \\ y & = & 7 & \leftarrow \textbf{Solve for } y. \end{array}$$

Since $x = 2$ and $y = 7$, the solution of the system is $(2, 7)$.

You will want a good example of solving systems by adding to eliminate one variable. Here is a good page to copy into your notes.

You can solve some systems of equations by adding.

Step 1: Eliminate one variable.

$$\begin{array}{rcll} 2x + 3y & = & 12 & \\ x - 3y & = & -3 & \\ \hline 3x + 0 & = & 9 & \leftarrow \text{Add} \\ x & = & 3 & \leftarrow \text{Solve for } x. \end{array}$$

Step 2: Substitute the value you found into one equation.

$$\begin{array}{rcll} 2x + 3y & = & 12 & \leftarrow \text{Write either equation.} \\ 2(3) + 3y & = & 12 & \leftarrow \text{Substitute 3 for } x. \\ 6 + 3y & = & 12 & \leftarrow \text{Simplify.} \\ 3y & = & 6 & \leftarrow \text{Divide by 3.} \\ y & = & 2 & \leftarrow \text{Solve for } y. \end{array}$$

The solution is $(3, 2)$.

❶ Solving a System by Adding

Solve the system by elimination.

$$2x + 3y = 18$$
$$5x - 3y = 3$$

Refer to your notes, and we will work through this problem as a whole group.

Step 1 The coefficients of ☐ are additive inverses, so add the equations to eliminate ☐.

$$2x + 3y = 18$$
$$+\ 5x - 3y = 3$$
$$\square = \square \quad \leftarrow \textbf{Add the two equations.}$$
$$x = \square \quad \leftarrow \textbf{Solve for } x.$$

Step 2 Substitute ☐ for x in either equation and solve for ☐.

$$2x + 3y = 18$$
$$2 \cdot \square + 3y = 18 \quad \leftarrow \textbf{Substitute } \square \textbf{ for } x.$$
$$\square + 3y = 18 \quad \leftarrow \textbf{Simplify.}$$
$$y = \square \quad \leftarrow \textbf{Solve for } y.$$

Check Replace x with ☐ and y with ☐ in each equation.

$2x + 3y = 18$

$2 \cdot \square + 3 \cdot \square \stackrel{?}{=} 18$

$\square = \square$ ✓

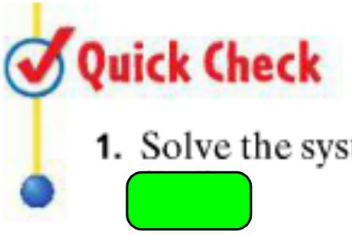
$5x - 3y = 3$

$5 \cdot \square - 3 \cdot \square \stackrel{?}{=} 3$

$\square = \square$ ✓

The solution of the system is ☐.

Table talk about this problem, then we will share/compare answers.



1. Solve the system by elimination.

$$3x + 4y = 16$$
$$-3x + 2y = 8$$

EXAMPLE 2 Solving a System by Subtracting

At the school store, Ricardo bought 4 pencils and 6 erasers and spent \$2.60. Annabelle bought 4 pencils and 10 erasers and spent \$3.80. Solve the system of equations to determine the cost of 1 pencil and the cost of 1 eraser.

Step 1 Write the system of equations. Let p = the cost of each pencil, and let e = the cost of each eraser.

$$\begin{aligned} 4p + 6e &= 2.60 \quad \leftarrow \text{Amount Ricardo spent} \\ 4p + 10e &= 3.80 \quad \leftarrow \text{Amount Annabelle spent} \end{aligned}$$

Step 2 Eliminate one variable. Since the coefficients of y are the same, subtract to eliminate p.

$$\begin{aligned} 4p + 6e &= 2.60 \\ -\ (4p + 10e &= 3.80) \\ \hline -4e &= -1.20 \quad \leftarrow \text{Subtract.} \\ e &= 0.30 \quad \leftarrow \text{Solve for } e. \end{aligned}$$

The cost of one eraser is \$0.30.

Step 3 Substitute 0.30 for e in either equation and solve for p.

$$\begin{aligned} 4p + 10e &= 3.80 \quad \leftarrow \text{Use either equation.} \\ 4p + 10(0.30) &= 3.80 \quad \leftarrow \text{Substitute 0.30 for } e. \\ 4p + 3 &= 3.80 \quad \leftarrow \text{Simplify.} \\ 4p &= 0.80 \quad \leftarrow \text{Subtract 3 from each side.} \\ p &= 0.20 \quad \leftarrow \text{Divide each side by 4.} \end{aligned}$$

The cost of one pencil is \$0.20.

This is a good example of solving systems with subtraction to eliminate. You will want to put all of this page in your notes.

You can solve some systems of equations by subtracting.

Step 1: Eliminate one variable.

$$\begin{aligned} 3a + 2b &= 3 \\ -(3a + b &= 0) \\ \hline b &= 3 \quad \leftarrow \text{Subtract.} \end{aligned}$$

Step 2: Substitute the value you found into one equation.

$$\begin{aligned} 3a + 2b &= 3 \quad \leftarrow \text{Write either equation.} \\ 3a + 2(3) &= 3 \quad \leftarrow \text{Substitute 3 for } b. \\ 3a + 6 &= 3 \quad \leftarrow \text{Simplify.} \\ 3a &= -3 \quad \leftarrow \text{Divide by 3.} \\ a &= -1 \quad \leftarrow \text{Solve for } a. \end{aligned}$$

The solution is $(-1, 3)$.

2 Solving a System by Multiplying Solve the system of equations by elimination.

Refer to your notes as we work through this problem as a whole group.

$$-6x + 4y = 2$$
$$-3x - 2y = -7$$

Step 1 Multiply both sides of the one equation by a number so that the coefficients of x are additive inverses of each other ☐.

☐ $\cdot (-3y - 2y) =$ ☐ $\cdot (-7)$ ← **Multiply the second equation by** ☐.

☐ $=$ ☐ ← **Simplify.**

Step 2 Eliminate one variable by combining equal expressions.

$-6x + 4y = 2$

$+$ ☐ ← **Add**

☐

$y =$ ☐ ← **Solve for *y*.**

Step 3 Substitute ☐ for y in either equation and solve for ☐.

$-6x + 4y = 2$

$-6x + 4 \cdot$ ☐ $= 2$ ← **Substitute** ☐ **for *y*.**

$-6x +$ ☐ $= 2$ ← **Simplify.**

$x =$ ☐ ← **Solve for *x*.**

Check Replace x with ☐ and y with ☐ in each equation.

$-6x + 4y = 2$

$-6 \cdot$ ☐ $+ 4 \cdot$ ☐ $\stackrel{?}{=} 2$

☐ $=$ ☐ ✓

$-3x - 2y = -7$

$-3 \cdot$ ☐ $- 2 \cdot$ ☐ $\stackrel{?}{=} -7$

☐ $=$ ☐ ✓

The solution of the system is ☐.

There are also ways to solve by multiplying. This will use your knowledge of factors in order to eliminate a variable.

EXAMPLE Solving a System By Multiplying

3 Solve the system by elimination.

$$10x - 7y = 2$$
$$-5x + 3y = -3$$

Step 1 To eliminate x, multiply each side of the second equation by 2. Then add.

$$10x - 7y = 2$$
$$2(-5x + 3y) = 2(-3)$$

$$10x - 7y = 2$$
$$+ \; -10x + 6y = -6$$
$$-1y = -4 \quad \leftarrow \textbf{Add.}$$
$$y = 4 \quad \leftarrow \textbf{Solve for } y.$$

Step 2 Substitute 4 for y in either original equation and solve for x.

$10x - 7y = 2$ ← **Use the first equation.**

$10x - 7(4) = 2$ ← **Substitute 4 for *y*.**

$10x - 28 = 2$ ← **Simplify.**

$10x = 30$ ← **Add 28 to each side.**

$x = 3$ ← **Divide each side by 10.**

The solution is $(3, 4)$.

You will also want a good example with multiplication in your notes. Copy this page.

You can solve some systems of equations by multiplying.

$3x + 2y = 14$
$-6x + 4y = 4$

Step 1: To eliminate x, multiply each side of the first equation by 2. Then add.

$2(3x + 2y) = 2(14)$ $\quad 6x + 4y = 28$

$-6x + 4y = 4$ $\quad -6x + 4y = 4$

$8y = 32$

$y = 4$

Step 2: Substitute 4 in either original equation and solve for x.

$3x + 2y = 14$ ← Use the first equation.

$3x + 2(4) = 14$ ← Substitute 4 for y.

$3x + 8 = 14$ ← Simplify.

$3x = 6$ ← Subtract 8 from each side.

$x = 2$ ← Divide each side by 3.

The solution is $(2, 4)$.

Example

❷ **Solving a System by Multiplying** Solve the system of equations by elimination.

$-6x + 4y = 2$
$-3x - 2y = -7$

Step 1 Multiply both sides of the one equation by a number so that the coefficients of x are additive inverses of each other [−2].

[−2] · (−3y − 2y) = [−2] · (−7) ← Multiply the second equation by [−2].

[6x + 4y] = [14] ← Simplify.

Step 2 Eliminate one variable by combining equal expressions.

$-6x + 4y = 2$
$+$ [6x + 4y = 14] ← Add

[8y = 16]

$y =$ [2] ← Solve for y.

Step 3 Substitute [2] for y in either equation and solve for [x].

$-6x + 4y = 2$

$-6x + 4 \cdot$ [2] $= 2$ ← Substitute [2] for y.

$-6x +$ [8] $= 2$ ← Simplify.

$x =$ [1] ← Solve for x.

Check Replace x with [1] and y with [2] in each equation.

$-6x + 4y = 2$ $\quad -3x - 2y = -7$

$-6 \cdot$ [1] $+ 4 \cdot$ [2] $\stackrel{?}{=} 2$ $\quad -3 \cdot$ [1] $- 2 \cdot$ [2] $\stackrel{?}{=} -7$

[2] = [2] ✓ $\quad$ [−7] = [−7] ✓

The solution of the system is [(1, 2)].

Name _____ Class _____ Date _____

5-3 • Guided Problem Solving

GPS Student Page 184, Exercise 11:

Writing in Math Explain how to solve the system below using the elimination method.

$$5x - 6y = 8$$
$$-3x + 11y = 10$$

Understand

1. What are you being asked to do?

2. How will you find the answer?

Plan and Carry Out

3. Multiply the first equation by 3.

4. Multiply the second equation by 5.

5. Add the equations to eliminate x.

6. Substitute the value for y. Solve for x.

7. What is the solution of the system? _____

Check

8. How can you check your answer?

Solve Another Problem

9. Explain how to solve the system below using the elimination method.

$$4x + 2y = 28$$
$$3x - 3y = 3$$

Name _____ Class _____ Date _____

Practice 5-3 Solving Systems by Elimination

Solve each system of equations by elimination. Check your solution.

1. $x - y = 5$
   $x + y = 11$

2. $9x + 4y = 7$
   $6x - 4y = 2$

3. $-3x + 6y = 15$
   $-3x - 2y = -1$

4. $-\frac{9}{4}x + 3y = 12$
   $\frac{1}{4}x - 6y = 10$

5. At the craft store, Lisa bought 3 clay blocks and 2 molds for $15.50. Tony bought 6 clay blocks and 1 mold for $19.00. Write and solve a system of equations to find the cost of each clay block and each mold.