

3-3

Solving Inequalities by
Multiplying or Dividing

© CONTENT STANDARDS

7.EE.4.b

New Vocabulary Division Property of Inequality, Multiplication Property of Inequality

Objective: I can solve inequalities by multiplying or dividing.

Why Learn This?

Inequalities can help you plan. You can solve inequalities to make sure you have enough ingredients when you are cooking.

Look at the pattern when you divide each side of an inequality by an integer.



$$18 > 12$$

$$\frac{18}{6} > \frac{12}{6}$$

$$\frac{18}{3} > \frac{12}{3}$$

$$\frac{18}{-2} < \frac{12}{-2}$$

$$\frac{18}{-6} < \frac{12}{-6}$$

← When the integer is positive, the direction of the inequality symbol stays the same.

← When the integer is negative, the direction of the inequality symbol is reversed.

KEY CONCEPTS**Division Property of Inequality**

If you divide each side of an inequality by the same positive number, the direction of the inequality symbol remains unchanged.

Arithmetic

$$9 > 6, \text{ so } \frac{9}{3} > \frac{6}{3}$$

$$15 < 20, \text{ so } \frac{15}{5} < \frac{20}{5}$$

Algebra

$$\text{If } a > b, \text{ and } c \text{ is positive, then } \frac{a}{c} > \frac{b}{c}.$$

$$\text{If } a < b, \text{ and } c \text{ is positive, then } \frac{a}{c} < \frac{b}{c}.$$

If you divide each side of an inequality by the same negative number, the direction of the inequality symbol is reversed.

Arithmetic

$$16 > 12, \text{ so } \frac{16}{-4} < \frac{12}{-4}$$

$$10 < 18, \text{ so } \frac{10}{-2} > \frac{18}{-2}$$

Algebra

$$\text{If } a > b, \text{ and } c \text{ is negative, then } \frac{a}{c} < \frac{b}{c}.$$

$$\text{If } a < b, \text{ and } c \text{ is negative, then } \frac{a}{c} > \frac{b}{c}.$$

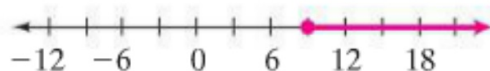
EXAMPLE**Solving Inequalities by Dividing**

- 1 Solve $-3y \leq -27$. Graph the solution.



$$-3y \leq -27$$

$$\frac{-3y}{-3} \geq \frac{-27}{-3} \quad \leftarrow \text{Divide each side by } -3. \text{ Reverse the direction of the symbol.}$$

$$y \geq 9 \quad \leftarrow \text{Simplify.}$$



Additional Examples

- 1 Solve $6a \leq -18$. Graph the solution. 
- 

Quick Check

1. Solve each inequality. Graph the solution.

a. $-4p < 36$

b. $-1.8m \geq -5.4$

c. $\frac{2}{3}n > -2$



You can solve an inequality that involves multiplication by dividing each side of the inequality by the same number.

EXAMPLE Application: Planning

- 2 Your class is taking a trip to a museum that is 190 miles away. The bus can travel at 55 miles per hour. At least how many hours should your class plan for the trip to the museum?



Words 55 times number of hours is at least total miles

Let h = the number of hours.

Expression $55 \cdot h \geq 190$

$$55h \geq 190$$
$$\frac{55h}{55} \geq \frac{190}{55} \quad \leftarrow \text{Divide each side by 55.}$$
$$h \geq 3.4545 \dots \quad \leftarrow \text{Simplify.}$$
$$h \geq 3.5 \quad \leftarrow \text{Round up to the nearest half hour.}$$

Your class should plan for at least 3 hours and 30 minutes.

- 2 A woodworker makes a profit of \$30 on each picture frame sold. How many frames must he sell to make a profit of at least \$500?

Quick Check

2. A long-distance telephone company is offering a special rate of \$.06 per minute. Your budget for long-distance telephone calls is \$25 for the month. At most how many minutes of long distance can you use for the month with this rate?

The properties of inequality apply to multiplication as well.

KEY CONCEPTS Multiplication Property of Inequality

If you multiply each side of an inequality by the same positive number, the direction of the inequality symbol remains unchanged.

Arithmetic

$$12 > 8, \text{ so } 12 \cdot 2 > 8 \cdot 2$$

$$3 < 6, \text{ so } 3 \cdot 4 < 6 \cdot 4$$

Algebra

If $a > b$, and c is positive,
then $a \cdot c > b \cdot c$.

If $a < b$, and c is positive,
then $a \cdot c < b \cdot c$.

If you multiply each side of an inequality by the same negative number, the direction of the inequality symbol is reversed.

Arithmetic

$$6 > 2, \text{ so } 6(-3) < 2(-3)$$

$$3 < 5, \text{ so } 3(-2) > 5(-2)$$

Algebra

If $a > b$, and c is negative,
then $a \cdot c < b \cdot c$.

If $a < b$, and c is negative,
then $a \cdot c > b \cdot c$.

EXAMPLE**Solving Inequalities by Multiplying**

3 Solve $\frac{y}{-8} \geq 2$.

$$\frac{y}{-8} \geq 2$$

$$-8 \cdot \frac{y}{-8} \leq -8 \cdot 2 \quad \leftarrow \begin{array}{l} \text{Multiply each side by } -8. \\ \text{Reverse the direction of the symbol.} \end{array}$$

$$y \leq -16 \quad \leftarrow \text{Simplify.}$$

 **Quick Check**

3. Solve $\frac{k}{-5} < -4$. Graph the solution.



EXAMPLE**Solving Inequalities by Multiplying by the Reciprocal**

4 Solve $-\frac{2}{3}h < -6$.

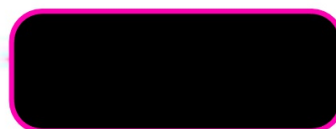
$$-\frac{2}{3}h < -6$$

$$-\frac{3}{2} \cdot \left(-\frac{2}{3}h > -\frac{3}{2} \cdot (-6)\right) \leftarrow \begin{array}{l} \text{Multiply each side by } -\frac{3}{2}, \text{ the reciprocal of} \\ -\frac{2}{3}. \text{ Reverse the direction of the symbol.} \\ \leftarrow \text{Simplify.} \end{array}$$

$$h > 9$$

**Quick Check**

4. Solve $-\frac{3}{4}w \geq 12$. Graph the solution.





Check Your Understanding

1. **Vocabulary** What happens when you multiply each side of an inequality by a negative number?

2. If $x > y$, which statement is NOT always true?

(A) $y < x$

(C) $x - z > y - z$

(B) $x + z > y + z$

(D) $xz > yz$

Fill in the missing inequality symbol.

3. $3m > 99$

$\frac{3m}{3} \blacksquare \frac{99}{3}$

4. $-1.6z \leq 80$

$\frac{-1.6z}{-1.6} \blacksquare \frac{80}{-1.6}$

5. $\frac{d}{-3} < 12$

$\frac{d}{-3} \cdot (-3) \blacksquare 12 \cdot (-3)$

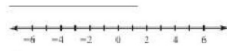
You have an assignment worksheet, due tomorrow.

Practice 3-3

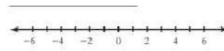
Solving Inequalities by Multiplying or Dividing

Solve each inequality. Graph each solution.

1. $6w \leq 36$



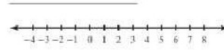
2. $10a \geq -40$



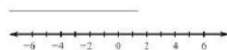
3. $\frac{f}{3} \leq -2$



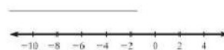
4. $\frac{x}{4} > 2$



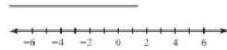
5. $7a > -28$



6. $-\frac{c}{3} \geq 3$



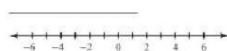
7. $\frac{f}{2} > -1$



8. $9a \leq 63$



9. $4w \geq -12$



10. $\frac{h}{2} \geq -5$



Write an inequality to solve each problem. Then solve the inequality.

11. Marcus wants to buy 5 baseballs. He has \$35. What is the most each baseball can cost?

12. Melinda charges \$4 per hour for babysitting. Mrs. Garden does not want to spend more than \$25 for babysitting. What is the maximum number of hours that she can have Melinda babysit?
