

9-5

Exploring Similar Solids

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8.G.9

What You'll Learn

To use proportions to find missing measurements of similar solids, including surface area and volume

 **New Vocabulary** similar solids

Why Learn This?

The dolls in the photo at the right have proportional heights and diameters. These and other art objects are examples of similar solids.

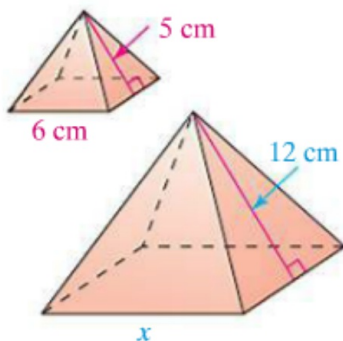
Two solids are **similar solids** if they have the same shape and if all of their corresponding dimensions are proportional.

You can use a proportion to find the unknown dimensions of similar solids.



EXAMPLE**Finding Dimensions of a Similar Solid**

1 The two pyramids below are similar. Find the value of x .



Use corresponding parts to write a proportion.

$$\frac{x}{6} = \frac{12}{5} \quad \leftarrow \begin{array}{l} \text{dimensions of large pyramid} \\ \text{dimensions of small pyramid} \end{array}$$

$$6 \cdot \frac{x}{6} = \frac{12}{5} \cdot 6 \quad \leftarrow \text{Multiply each side by 6.}$$

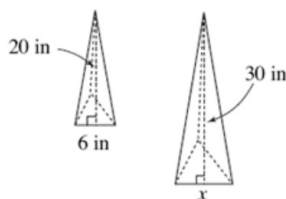
$$x = \frac{72}{5} \quad \leftarrow \text{Simplify.}$$

$$= 14.4$$

The base-edge length x is 14.4 cm.

EXAMPLE

The two triangular pyramids are similar. Find the value of x .



Use corresponding parts to write a proportion.

$$\frac{X}{6} = \frac{30}{20}$$

$$6 \cdot \frac{X}{6} = \frac{30}{20} \cdot 6$$

$$X = \frac{180}{20}$$

$$= 9$$

The base-edge length x is 9 in.

Example

- 1 **Finding Dimensions of a Similar Solid** At an ice cream shop, the small and large cones are similar. The small cone has a radius of 3 cm and a height of 12 cm. The large cone has a radius of 5 cm. What is the height of the large ice cream cone? Let h = the height of the large cone. Use corresponding parts to write a proportion.

$$\frac{h}{12} = \frac{5}{3}$$

← dimensions of large cone
← dimensions of small cone

$$12 \times \frac{h}{12} = \frac{5}{3} \times 12$$

← Multiply each side by 12.

$$h = \frac{60}{3}$$

← Simplify.

$$= 20$$

The height of the large cone is **20 cm**.

Quick Check

1. Two cylinders are similar. The small cylinder has a diameter of 4 m and a height of h . The large cylinder has a diameter of 5 m and a height of 11 m. What is the value of h ?

KEY CONCEPTS**Surface Area and Volume of Similar Solids**

If the ratio of the corresponding dimensions of similar solids is $\frac{a}{b}$, then

- the ratio of surface areas is $\frac{a^2}{b^2}$ and
- the ratio of volumes is $\frac{a^3}{b^3}$.

You can use ratios of corresponding dimensions to find the surface area and volume of similar solids.

EXAMPLE**Surface Area and Volume of Similar Solids**

- 2 Pottery** The surface area of the cylindrical pitcher below is about 90 in.². Its volume is about 157 in.³. The pitcher and creamer are similar. Find the surface area and volume of the creamer.



Careers Some potters make ceramic pieces by shaping wet clay as it spins on a potter's wheel.



The ratio of the diameters is $\frac{3}{6}$, or $\frac{1}{2}$. So the ratio of the surface areas is $\frac{1^2}{2^2}$, or $\frac{1}{4}$.

$$\frac{\text{surface area of creamer}}{\text{surface area of pitcher}} = \frac{1}{4} \quad \leftarrow \text{Write a proportion.}$$

$$\frac{S}{90} = \frac{1}{4} \quad \leftarrow \text{Substitute the surface area of the pitcher.}$$

$$4 \cdot S = 1 \cdot 90 \quad \leftarrow \text{Write the cross products.}$$

$$S = 22.5 \quad \leftarrow \text{Simplify.}$$

The surface area of the creamer is about 22.5 in.².

If the ratio of the diameters is $\frac{1}{2}$, the ratio of the volumes is $\frac{1^3}{2^3}$, or $\frac{1}{8}$.

$$\frac{\text{volume of creamer}}{\text{volume of pitcher}} = \frac{1}{8} \quad \leftarrow \text{Write a proportion.}$$

$$\frac{V}{157} = \frac{1}{8} \quad \leftarrow \text{Substitute the volume of the pitcher.}$$

$$8 \cdot V = 1 \cdot 157 \quad \leftarrow \text{Write the cross products.}$$

$$V = 19.625 \quad \leftarrow \text{Simplify.}$$

The volume of the creamer is about 20 in.³.

2 EXAMPLE The surface area of a cylindrical vase is about 1,470 in.² and its volume is about 490 in.³. The vase is similar to a drinking glass. The diameter of the vase is 6 in. and that of the drinking glass is 3 in. Find the surface area and volume of the drinking glass.

The ratio of the diameters is $\frac{3}{6}$, or $\frac{1}{2}$. So the ratio of the surface areas is $\frac{1^2}{2^2}$, or $\frac{1}{4}$.

$$\frac{\text{surface area of drinking glass}}{\text{surface area of cylindrical vase}} = \frac{1}{4} \quad \leftarrow \text{Write a proportion.}$$

$$\frac{s}{1,470} = \frac{1}{4} \quad \leftarrow \text{Substitute the surface area of the glass.}$$

$$4 \cdot s = 1 \cdot 1,470 \quad \leftarrow \text{Write the cross products.}$$

$$s = 367.5 \quad \leftarrow \text{Simplify.}$$

The surface area of the drinking glass is about 368 in.².
If the ratio of the diameters is $\frac{1}{2}$, the ratio of the volumes is $\frac{1^3}{2^3}$ or $\frac{1}{8}$.

$$\frac{\text{volume of drinking glass}}{\text{volume of cylindrical vase}} = \frac{1}{8} \quad \leftarrow \text{Write a proportion.}$$

$$\frac{v}{490} = \frac{1}{8} \quad \leftarrow \text{Substitute the volume of the glass.}$$

$$8 \cdot v = 1 \cdot 490 \quad \leftarrow \text{Write the cross products.}$$

$$v = 61.25 \quad \leftarrow \text{Simplify.}$$

The volume of the drinking glass is about 61 in.³.

Example

2 Surface Area and Volume of Similar Solids A square pyramid has a surface area of 39 cm² and a volume of 12 cm³. The pyramid is similar to a larger pyramid, but its side length is $\frac{2}{3}$ that of the larger pyramid. Find the surface area and volume of the larger pyramid.

The ratio of the side lengths is $\frac{2}{3}$, so the ratio of the surface areas is $\frac{2^2}{3^2}$, or $\frac{4}{9}$.

$$\frac{\text{surface area of the small pyramid}}{\text{surface area of the large pyramid}} = \frac{4}{9} \quad \leftarrow \text{Write a proportion.}$$

$$\frac{39}{S} = \frac{4}{9} \quad \leftarrow \text{Substitute the surface area of the small pyramid.}$$

$$39 \cdot 9 = S \cdot 4 \quad \leftarrow \text{Write the cross products.}$$

$$S = 87.75 \quad \leftarrow \text{Simplify.}$$

The surface area of the large pyramid is **87.75 cm²**.

If the ratio of the side lengths is $\frac{2}{3}$, the ratio of the volumes is $\frac{2^3}{3^3}$, or $\frac{8}{27}$.

$$\frac{\text{volume of the small pyramid}}{\text{volume of the large pyramid}} = \frac{8}{27} \quad \leftarrow \text{Write a proportion.}$$

$$\frac{12}{V} = \frac{8}{27} \quad \leftarrow \text{Substitute the volume of the small pyramid.}$$

$$12 \cdot 27 = V \cdot 8 \quad \leftarrow \text{Write the cross products.}$$

$$V = 40.5 \quad \leftarrow \text{Simplify.}$$

The volume of the large pyramid is **40.5 cm³**.

9-5 • Guided Problem Solving

Student Page 322, Exercise 14:

Carpentry Gina used 78 square feet of plywood to build a storage bin to hold her gardening supplies. How much plywood will she need to build a similar box for her hand tools if the dimensions of the box are half the dimensions of the bin?

Understand

1. What are you asked to do?

2. How many square feet of plywood did Gina use? _____

Plan and Carry Out

3. What is the ratio for the surface area of similar solids? _____
4. What are the dimensions of the similar box?

5. What is the ratio of the surface areas? _____
6. Write a proportion. _____
7. Solve. _____

Check

8. How can you tell if your answer is reasonable?

Solve Another Problem

9. In pottery class, Mark made a small cylindrical bowl with a volume of 75 in.^3 and a radius of 2 inches. He also made a larger bowl with a similar shape. It has a diameter of 8 inches. Find the volume of the larger bowl.

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