7-5 Proving Triangles Similar

**What You'll Learn**

To determine measures of the angles of triangles and use them to help prove that triangles are similar

© **CONTENT STANDARDS**

8.G.5

You will need Evernote today.

✓ **Check Skills You'll Need**

1. **Vocabulary Review**
   If two triangles are *similar*, then their corresponding angles are __?__ and their corresponding side lengths are __?__ .

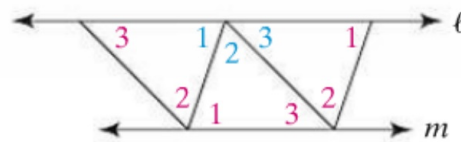2. Is a square with a side length of 6 in. similar to a square with a side length of 10 in.? Explain.
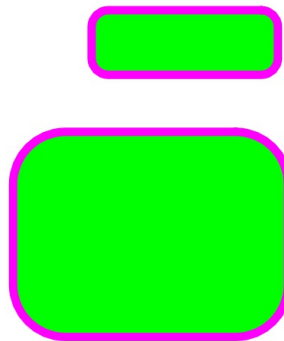
Why Learn This?

Architects and engineers often make use of triangles when designing buildings and other structures.

Three copies of the same triangle have been arranged as shown. Lines ℓ and m are parallel because their alternate interior angles are congruent.

The angles labeled in blue show that $\angle 1$, $\angle 2$, and $\angle 3$ form a straight angle along line ℓ, which means that the sum of their measures is $180°$. Therefore, the sum of the measures of the angles of each of the triangles is $180°$.

please type this in your notes

KEY CONCEPTS   **Angle Sum of a Triangle**

The sum of the measures of the angles of any triangle is 180°.

EXAMPLE   **Finding an Angle Measure**

1 **Multiple Choice** $\triangle RST$ forms part of the front of a cabin as shown at the left. What is the measure of $\angle S$?

Ⓐ 55°   Ⓑ 70°   Ⓒ 125°   Ⓓ 110°

$$m\angle R + m\angle S + m\angle T = 180°$$ ← Angle sum of a triangle
$$55° + m\angle S + 55° = 180°$$ ← Substitute.
$$m\angle S + 110° = 180°$$ ← Simplify.
$$m\angle S + 110° - 110° = 180° - 110°$$ ← Subtract 110° from each side.
$$m\angle S = 70°$$ ← Simplify.

❶ Finding an Angle Measure △BAC forms part of a bridge truss.
What is the measure of ∠C?

$m\angle B +$ [____] $+ m\angle C = 180°$ ← Angle sum of a triangle.

[____] $+$ [____] $+ m\angle C = 180°$ ← Substitute.

$m\angle C +$ [____] $= 180°$ ← Simplify.

$m\angle C + 100° -$ [____] $= 180° -$ [____] ← Subtract 100° from each side.

$m\angle C =$ [____] ← Simplify.

(Triangle with vertex C at top, B at bottom left with 60°, A at bottom right with 40°.)
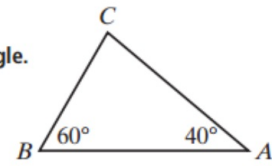
Quick Check

1. What is the measure of ∠E in △DEF?

[____]

(Right triangle with vertex E at top, right angle at D bottom left, F at bottom right with 24°.)
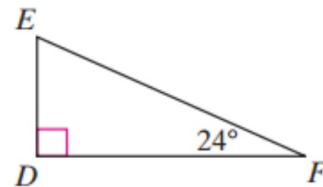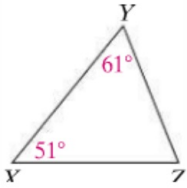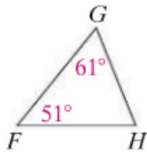
If all pairs of corresponding angles of two triangles are congruent, then the triangles have the same shape and the triangles are similar. In the diagram at the left, $\angle F \cong \angle X$ and $\angle G \cong \angle Y$.

What about the third pair of corresponding angles? By using the angle sum of a triangle, you can determine that $m\angle H = 68°$ and $m\angle Z = 68°$. Because all three pairs of corresponding angles are congruent, $\triangle FGH \sim \triangle XYZ$.

This example shows that if two pairs of corresponding angles of two triangles are congruent, then the third pair of corresponding angles must also be congruent and the triangles are similar.

Capture this picture for your notes

KEY CONCEPTS Angle-Angle (AA) Similarity

If two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.
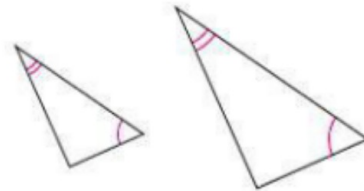
EXAMPLE · Similar Triangles

2 Show that the pair of triangles is similar.

Step 1 Use the angle sum of a triangle to find $m\angle R$.

$$91° + 37° + m\angle R = 180°$$
$$128° + m\angle R = 180°$$
$$128° - 128° + m\angle R = 180° - 128°$$
$$m\angle R = 52°$$

Step 2 Use AA similarity.

$\angle P \cong \angle L$ ← **Each measures 91°.**

$\angle R \cong \angle K$ ← **Each measures 52°.**

$\triangle PQR \sim \triangle LMK$ by AA similarity.
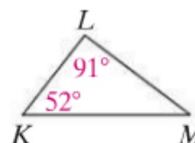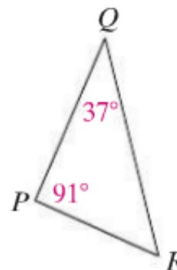
Test Prep Tip

There is often more than one way to show that two triangles are similar. For example, you could have started by finding $m\angle M$ instead of $m\angle R$.

Example

2 Similar Triangles Show that the pair of triangles is similar.

Step 1 Use the angle sum of a triangle to find $m\angle R$.

$$89° + 43° + m\angle R = \boxed{}$$
$$132° + m\angle R = \boxed{}$$
$$132 - \boxed{} + m\angle R = \boxed{} - 132°$$
$$m\angle R = \boxed{}$$

Step 2 Use AA similarity.

$\angle P \cong \boxed{}$ ← **Each measures 89°.**

$\angle R \cong \boxed{}$ ← **Each measures 48°.**

$\triangle PQR \sim \triangle LMK$ by Angle-Angle similarity.
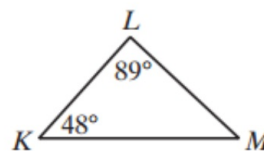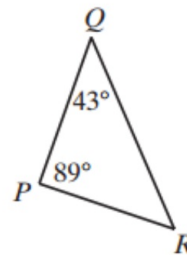
2. Show that each pair of triangles is similar.

a.

M

T   U
75°   75°

30°

75°
L   N   S

b.

B

C
88°

64°

28°
A   E   D
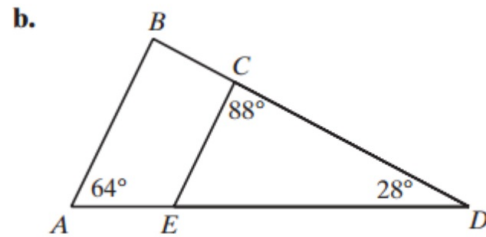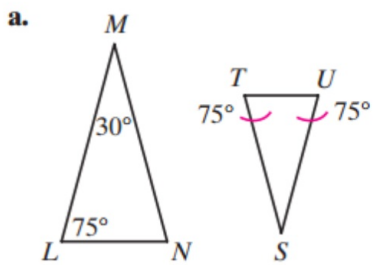
Check Your Understanding

For each triangle, find $m\angle 1$.

1.
74°

1

68°

2.
21°

51°

1

3.
37°

1
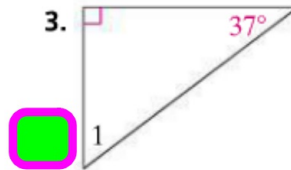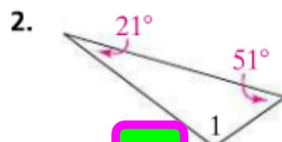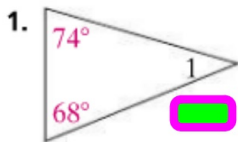
Use the triangles below for Exercises 4 and 5.

D   E
66°

63°
F

S
63°

66°
R   T

4. List the congruent corresponding angles of the two triangles.

5. Error Analysis Daisy writes $\triangle DEF \sim \triangle TRS$ by AA similarity. Erika writes $\triangle DEF \sim \triangle RST$ by AA similarity. Who is incorrect? Explain.
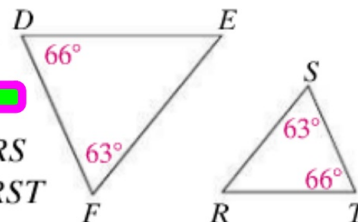
Vocabulary and Key Concepts

The sum of the measure of the angles of any triangle is ▮▮▮▮▮▮ _____.

If two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are ▮▮▮▮▮▮▮▮ _____.

7-5 • Guided Problem Solving

Proving Triangles Similar

GPS Student Page 235, Exercise 13:

City workers are laying out the paths in a new park, as shown in the diagram. Do the workers have enough information to determine $m\angle Q$? If so, explain how to find its measure. If not, explain why not.
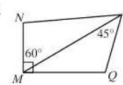
Understand

1. What are you being asked to find?
   _____

2. What is the angle measure of a right angle?
   _____

3. How many pairs of corresponding angles must be congruent in order for two triangles to be similar?
   _____

Plan and Carry Out

4. What angle measures does the diagram provide?
   $\angle NMQ =$ _____ $\angle NMP =$ _____ $\angle MPQ =$ _____

5. Based on the angle measures shown in the diagram, you can find $m\angle PMQ$. $\angle PMQ = \angle NMQ - \angle NMP =$ _____

6. Now that you know the measure of two angles in $\triangle MQP$, use angle sum of a triangle to find $\angle Q$. $180° - (\angle PMQ + \angle MPQ) = \angle Q =$ _____

7. Is $\triangle MQP \sim \triangle PNM$? How do you know?
   _____
   _____

Check

8. Given the angle measure you found for $\angle Q$, what is the angle sum of $\triangle MQP$? Show each angle measure _____

Solve Another Problem

9. For part of a theater set, students are supposed to build two similar triangular wood frames. The first frame has the following side/angle/side measure: 20 cm/80°/24 cm. The second frame has a different side-angle-side measures: 15 cm/80°/18 cm. Are the frames similar triangles? Explain.
   _____
   _____