Algebra

4-7 Proportional Relationships

CONTENT STANDARDS
7.RP.2.a, 7.RP.2.b, 7.RP.2.c, 7.RP.2.d

What You'll Learn

To identify proportional relationships and find constants of proportionality

New Vocabulary constant of proportionality

Why Learn This?

You know the total distances a cyclist traveled at different times during a bike-a-thon.

You can display the times and distances in a table or graph. Then you can determine if the times and distances have a proportional relationship. If they do, you can use the data to find the average speed at which the cyclist rode.

Examples

1 Using a Table to Determine a Proportional Relationship The table below shows the number of times Linda skipped rope in minutes during a fundraiser. Is there a proportional relationship between time and skips?

Minutes	0	5	12	15	17
Skips	0	150	360	450	510

Compare the ratios of time and rope skips.

rope skips → $\frac{150}{5} = \frac{\boxed{360}}{12} = \frac{450}{\boxed{15}} = \frac{\boxed{510}}{\boxed{17}}$
time →

The ratios are **equivalent**, so there is a **proportional** relationship between time and **rope skips**.

Keisha

Hours	0	2	4	5	7
Miles	0	13	26	32.5	45.5

EXAMPLE **Using a Table to Determine a Proportional Relationship**

Interpreting Data The table at the left shows the distances Keisha traveled during a bike-a-thon. Is there a proportional relationship between time and distance?

Compare the ratios of distance and time.

distance → $\frac{13}{2} = \frac{26}{4} = \frac{32.5}{5} = \frac{45.5}{7}$
time →

The ratios are equivalent, so there is a proportional relationship between time and distance.

Quick Check

Table talk about this problem.

1. The table shows the distances Dave rode in a bike-a-thon. Is there a proportional relationship? Explain. [redacted]

Dave

Hours	0	3	6	8	9
Miles	0	18.6	35.2	49.6	56.8

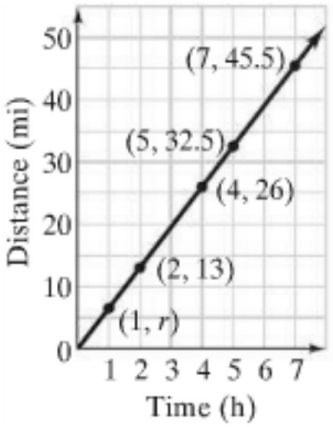
The graph of a proportional relationship is a straight line through the origin $(0, 0)$. The point $(1, r)$ on the graph of any proportional relationship represents the unit rate.

EXAMPLE 2 Using a Graph to Find a Unit Rate

The graph at the right displays the data given in Example 1. What is Keisha's speed in miles per hour?

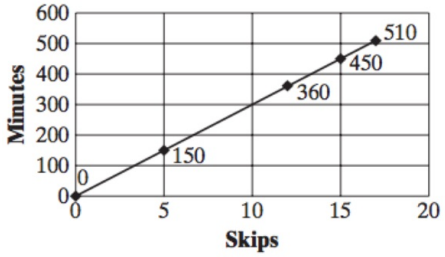
Keisha's speed is a unit rate. Find the value of r in the ordered pair $(1, r)$.

The line passes through $(0, 0)$ and $(2, 13)$. So, it must also pass through $(1, 6.5)$. Since $r = 6.5$, the unit rate is 6.5 mi/h. Keisha's speed is 6.5 mi/h.



Minutes	0	5	12	15	17
Skips	0	150	360	450	510

Using a Graph to Find a Unit Rate The graph below displays the data given in Example 1. What is Linda's speed in skips per minute?



Linda's speed is a unit rate. Find the value of r in the ordered pair $(1, r)$.

The graph of this relationship passes through $(0, 0)$ and $(\boxed{5}, 150)$. So, it must also pass through $(1, \boxed{30})$. Since $r = \boxed{30}$, the unit rate is $\boxed{30}$ $\boxed{\text{skips}}$ per $\boxed{\text{minute}}$.
Linda's speed is $\boxed{30}$ $\boxed{\text{skips}}$ per $\boxed{\text{minute}}$.

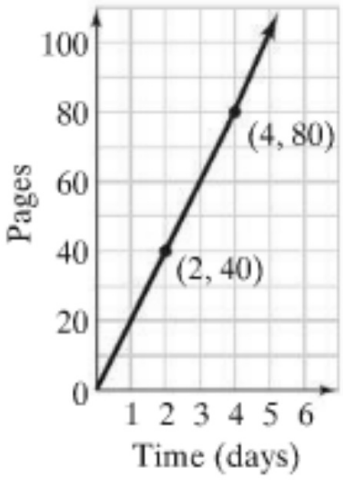


Table talk about this problem.

Quick Check

2. Use the graph at the left. What is Damon's reading speed in pages per day? [redacted]

The value of the ratio of quantities in a proportional relationship is called the **constant of proportionality**. This value is also equivalent to the unit rate.

EXAMPLE 3 Using a Ratio to Identify a Unit Rate

The table at the left shows a proportional relationship between the number of minutes and the amount the customer pays for cell phone service. Identify the constant of proportionality.

Minutes, m	Price, p (dollars)
100	$10
500	$50
1,000	$100
1,500	$150

Step 1 Use one data point to find the constant of proportionality c.

$\frac{\text{price}}{\text{minutes}} = \frac{10}{100}$ ← Find the price per minute by dividing the price by the number of minutes.

$= 0.1$ ← Simplify.

Step 2 Check by multiplying c times the first quantity.

$100 \times 0.1 = 10$ ✓ $\quad 500 \times 0.1 = 50$ ✓

$1{,}000 \times 0.1 = 100$ ✓ $\quad 1{,}500 \times 0.1 = 150$ ✓

The constant of proportionality is 0.1. The unit rate is $.10 per minute.

Now that you know the unit rate, you can write an equation that represents the proportional relationship. (Think cost per minute!)

Step 3 Use the constant of proportionality to write an equation to find the price p for m minutes. $p = 0.1\,m$

Using a Ratio to Identify a Unit Rate The table below shows a proportional relationship between the number of songs downloaded on a music site and the amount the customer pays. Identify the constant of proportionality.

Songs Downloaded, s	Price, p (dollars)
20	$10
40	$20
100	$50
120	$60

Step 1 Use one data point to find the constant of proportionality c.

$\frac{\text{price}}{\text{songs}} = \frac{\boxed{10}}{\boxed{20}}$ ← Find the price per song by **dividing** the **price** by the number of **songs**.

$= \boxed{0.5}$ ← Simplify.

Step 2 Check by multiplying c times the first quantity.

$20 \times \boxed{0.5} = \boxed{10}$ $\quad 40 \times \boxed{0.5} = \boxed{20}$

$100 \times \boxed{0.5} = \boxed{50}$ $\quad 120 \times \boxed{0.5} = \boxed{60}$

The constant of proportionality is **0.5**. This unit rate represents a payment of **$.50** per song.

Table talk about these.

Quick Check

3. Find the constant of proportionality for each table at the left.
   a. yards of cloth per blanket **b.** pay per hour

a.

Yards (y)	16	32	40
Blankets (b)	8	16	20

[redacted]

b.

Hours (h)	2	10	16
Pay (p)	$11	$55	$88

[redacted]

First try on your own, then compare answers with your table partners.

Find the constant of proportionality for each table of values.

10. profit per shirt sold

Shirts	5	10	15
Profit	$7.50	$15.00	$22.50

[redacted]

11. price per pound

Apples (lb)	4	5	6
Price	$7.96	$9.95	$11.94

[redacted]

Check Your Understanding

1. **Vocabulary** How is a constant of proportionality like a unit rate?

2. **Number Sense** The graph of a proportional relationship passes through the point $(2, 8)$. What is the constant of proportionality for this relationship?

Answer each question about the data in the table.

Bags	3	8	11
Dog Food (lb)	7.5	20	27.5

3. Is there a proportional relationship between bags and pounds of dog food? Explain your reasoning.

4. What is the constant of proportionality?

5. Using the constant of proportionality, what equation relates p pounds of dog food to b bags?

You have an assignment worksheet, due tomorrow.

You can use a calculator on this worksheet.

Name _____ Class _____ Date _____

Reteaching 4-7

Proportional Relationships

You can use a table to determine if there is a proportional relationship.

Compare the ratios to see if there is a proportional relationship.

Hours	2	4	5
Pages	12	24	30

$\frac{12}{2} = \frac{24}{4} = \frac{30}{5}$ ← There is a proportional relationship between pages and hours.

Hours	2	3	4
Pages	10	14	24

$\frac{10}{2} \neq \frac{14}{3} \neq \frac{24}{4}$ ← There is not a proportional relationship between pages and hours.

You can use a graph to find the unit rate.

The line passes through (0, 0) and (2, 12), therefore it passes through (1, 6).

Since $r = 6$, then the unit rate is 6 pages/hour.

You can use a ratio to find the unit rate.

The value of the ratio of quantities in a proportional relationship is called the **constant of proportionality**, which is equivalent to the unit rate.

$\frac{\text{PAGES}}{\text{HOUR}} = \frac{12}{2}$ ← **Find the pages per hour by dividing the number of pages by the number of hours.**

$= 6$ ← **Simplify.**

The constant of proportionality is 6. The unit rate is 6 pages/hour.

Determine whether each table represents a proportional relationship. If so, find the constant of proportionality.

1.

x	4	5	7
y	$10.40	$13.00	$17.50

2.

x	3	4	5
y	$\frac{21}{8}$	7	$\frac{35}{4}$