

**What You'll Learn**

To identify nonlinear functions

🔊 **New Vocabulary** nonlinear function, quadratic function, parabola

Copy these definitions into your notebooks:

**nonlinear**: If data does not form a straight line when graphed then its change is nonlinear.

**quadratic function**: the greatest power of the variable is 2, so look for  $x^2$ .

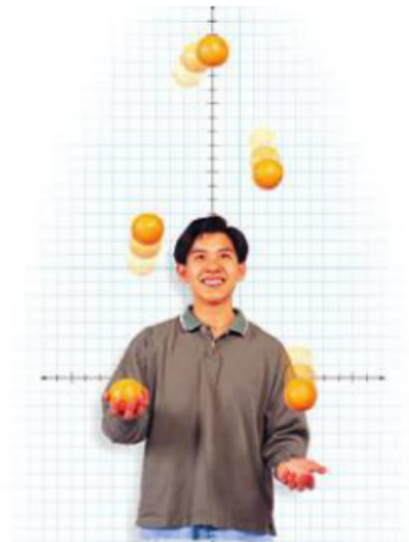
**parabola**: The graph of a quadratic function is a U-shaped or upsidedown U shaped curve, called a parabola.

## Why Learn This?

Think of a juggler's ball being tossed up in the air and falling back down. As the ball rises, its speed decreases. Then it falls, and the speed increases. The relationship between speed and time is not always the same.

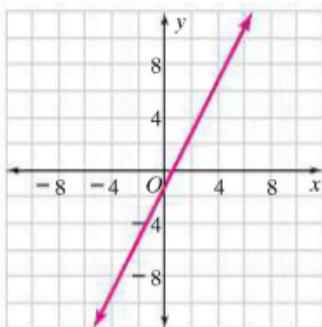
In cases like this, **nonlinear functions** are needed to describe how variables are related. Nonlinear functions are functions whose graphs are not straight lines. One example of a nonlinear function is a **quadratic function** in which the greatest exponent of a variable is 2.

The graph of a quadratic function is a U-shaped curve called a **parabola**. The curve may open upward or downward. When you throw a ball into the air, the path it follows is a parabola.

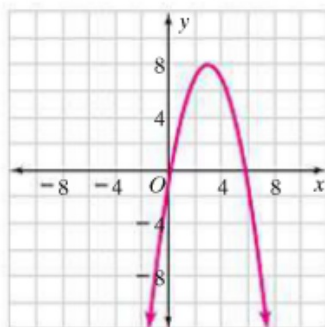


## EXAMPLE Nonlinear Functions From Graphs

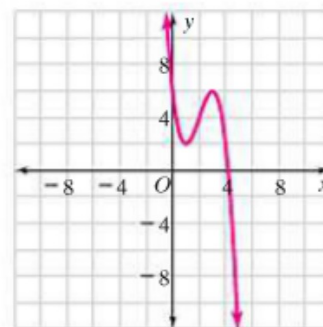
1 Which functions appear to be nonlinear?



Function 1



Function 2

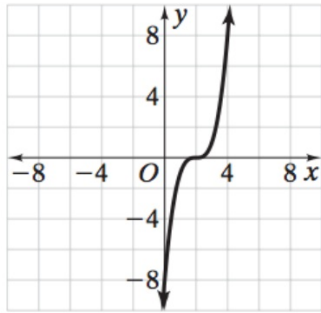


Function 3

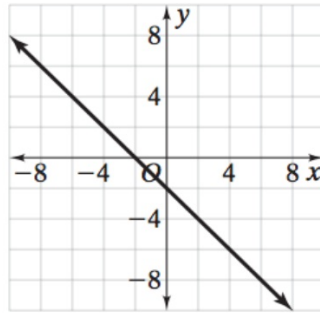
The graph of function 1 is a straight line and the graph of functions 2 and 3 are not straight lines. Functions 2 and 3 are nonlinear.

## Examples

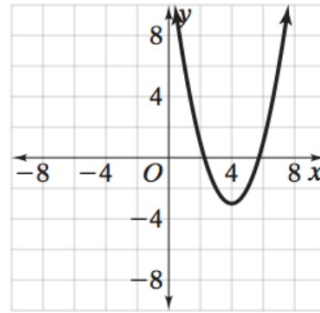
1 **Nonlinear Functions from Graphs** Which functions appear to be nonlinear?



Function 1



Function 2



Function 3

The graph of function 1 \_\_\_\_\_ a straight line.

The graph of function 2 \_\_\_\_\_ a straight line.

The graph of function 3 \_\_\_\_\_ a straight line.

So, function(s) \_\_\_\_\_ is/are nonlinear.

Copy this into your notes please.

You can identify nonlinear functions from equations and tables.

**Equation:** If a function's greatest exponent is 2 or greater, or if it has a variable for an exponent, the function is nonlinear.

**Table:** If the ratios between the changes in variables in a table are *not* the same, then the function is nonlinear.

## EXAMPLE

### Nonlinear Functions from Tables and Equations

2 Determine which function is nonlinear.

a.  $y = 4x^3 - 7$

$y = 8(x + 1) - 12$

$y = 8x + 8 - 12$

$y = 8x - 4$

$y = 4x^3 - 7$ : greatest exponent is 3     $y = 8x^1 - 4$ : greatest exponent is 1

The nonlinear function is  $y = 4x^3 - 7$ .

b.

x	2	4	6	8
y	10	7	4	1

Changes in x: +2, +2, +2  
Changes in y: -3, -3, -3

The ratio between the changes in variables is  $-\frac{2}{3}$ .

The ratios are the same.

The second table represents a nonlinear function.

x	1	2	3	4
y	1	8	27	64

Changes in x: +1, +1, +1  
Changes in y: +7, +19, +37

The ratios between the changes in variables are  $\frac{1}{7}$ ,  $\frac{1}{19}$ , and  $\frac{1}{31}$ .

The ratios are *not* the same.

2 **Nonlinear Functions from Tables and Equations** Determine which function is nonlinear.

a. Simplify each equation if possible.

$y = 3(x + 4) - 5$

$y = 2x^3 + 5$

$y = 3x + 7$

The greatest exponent in the first function is 1.

The greatest exponent in the second function is 3.

The second function is nonlinear because its greatest exponent is 2 or greater.

b. Find the ratios between the changes in variables for each table.

x	5	7	10	14
y	2	4	6	8

1,  $\frac{3}{2}$ , and 2

x	3	6	9	12
y	5	3	1	-1

$-\frac{3}{2}$

The ratios are not the same in the first table, so it represents a nonlinear function.

Sometimes you can determine if a function is nonlinear by its description.

### EXAMPLE

### Nonlinear Functions from Descriptions



3 Decide if the function described is linear or nonlinear.

A single cell divides to form two cells after 1 minute. All cells formed after this also divide in two every minute. A function relates time in minutes to the total number of cells.

As time increases by 1 minute, the total number of cells double which is not a constant rate of change. The function is nonlinear.

3 **Nonlinear Functions from Descriptions** Decide if the function described is linear or nonlinear.

Samantha throws a softball into the air. A function relates the height of the ball above the ground to time.

Decide if the height per second of the ball changes at a constant rate. Explain.

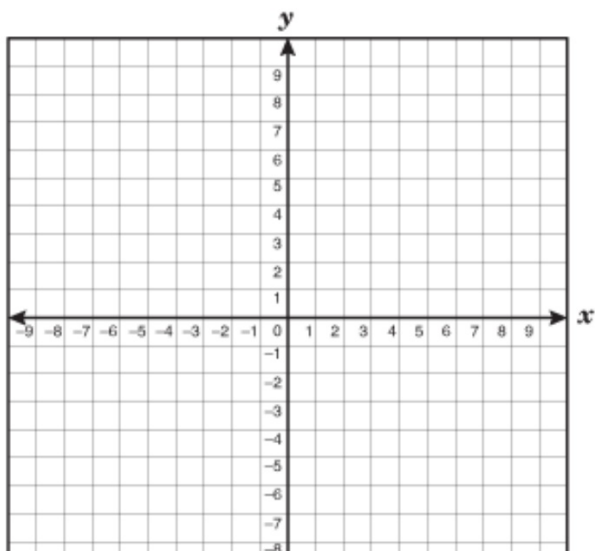
**No; it increases and then decreases.**

So, this function is **nonlinear**.

## Quick Check

1. Sketch the function that passes through the points in the table. Does the function appear to be nonlinear?

$x$	2	4	6	8
$y$	4	8	12	16



2. Which of the two given functions is nonlinear?

a.  $y = 17 - 4^x$

$y = 4 + 2(x + 7)$

b. 

$x$	3	6	9	12
$y$	3	5	7	10

$x$	6	7	8	9
$y$	-2	1	4	7

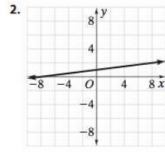
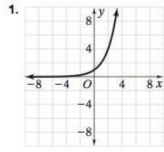
3. John mows lawns. He charges his customers \$15 per hour. A function relates the number of hours John works and the amount he earns. Is the function nonlinear? Explain.



The types of questions we just went through are exactly like the questions on your assignment worksheet.

**Practice 3-5** Nonlinear Functions

Identify each function as linear or nonlinear.



Circle the function in each pair that is nonlinear.

- |                                       |                        |                     |
|---------------------------------------|------------------------|---------------------|
| 3. $y = 2x$                           | 4. $y = 2x^3 + 7x - 1$ | 5. $y = 2(x - 3.5)$ |
| $y = x^2 - 4x + 6$                    | $y = 5x + 3$           | $y = 0.6x^4 + 2$    |
| 6. $y = \frac{2}{3}x^5 - 4x^3 + 5$    | 7. $y = 4^x$           | 8. $y = 2.4(5 - x)$ |
| $y = \frac{3}{5}x - 5 + \frac{2}{5}x$ | $y = 4x - 1 + 2x$      | $y = 3 + 5x - 2x^2$ |

9.

x	1	6	11	17
y	21	-2	-6	-10

10.

x	-2	-3	-4	-5
y	20	23	26	29

x	-3	-1	1	3
y	6	5	4	3

x	4	8	12	16
y	5	10	20	40

Determine if the function described is linear or nonlinear. Explain.

11. **Physics** Gravity causes an object to fall from a tall building. A function relates the object's speed while falling and time. \_\_\_\_\_
12. **Transportation** A train is traveling at a rate of 80 mi/hr. A function relates the distance the train has traveled to its rate of speed. \_\_\_\_\_