1-7 Multiplying Rational Numbers

What You'll Learn

To use number lines and properties to understand multiplication of rational numbers and to multiply rational numbers

CONTENT STANDARDS
7.NS.2, 7.NS.2.a, 7.NS.2.c, 7.NS.3

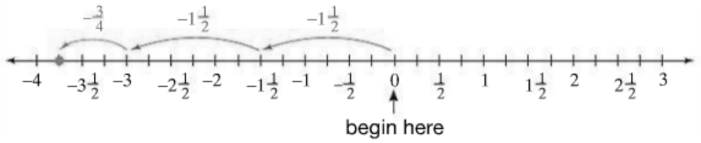
Why Learn This?

You can multiply rational numbers to represent situations with money.

Suppose you borrow \$8.75 from your parents every week for 8 weeks to cover bus fare and have not yet repaid them. You can calculate your debt to your parents by multiplying -8.75×8.

KEY CONCEPTS Multiplying Rational Numbers

The number line below shows the product $\left(2\frac{1}{2}\right) \cdot \left(-1\frac{1}{2}\right)$.



The Distributive Property explains why $2\frac{1}{2} \cdot \left(-1\frac{1}{2}\right)$ is equal to $2 \cdot \left(-1\frac{1}{2}\right) + \frac{1}{2} \cdot \left(-1\frac{1}{2}\right)$.

The number line above illustrates $2 \cdot \left(-1\frac{1}{2}\right) + \frac{1}{2} \cdot \left(-1\frac{1}{2}\right)$, because there are 2 groups of $\left(-1\frac{1}{2}\right)$ and $\frac{1}{2}$ group of $\left(-1\frac{1}{2}\right)$.

The sum of $2\frac{1}{2}$ groups of $-1\frac{1}{2}$ is $-3\frac{3}{4}$, and is equal to the product $\left(2\frac{1}{2}\right) \cdot \left(-1\frac{1}{2}\right)$, so $\left(2\frac{1}{2}\right) \cdot \left(-1\frac{1}{2}\right) = 3\frac{3}{4}$.

The rules for multiplying rational numbers are the same as those for multiplying integers.

EXAMPLE 1 Multiplying Positive Rational Numbers

When both factors are positive, the product is positive.

Find $\left(4\frac{3}{4}\right)\left(2\frac{1}{2}\right)$.

$$\left(4\tfrac{3}{4}\right)\left(2\tfrac{1}{2}\right) = \left(4\tfrac{3}{4}\right)\left(2 + \tfrac{1}{2}\right) \quad \leftarrow \text{Write } 2\tfrac{1}{2} \text{ as a sum.}$$

$$= \left(4\tfrac{3}{4} \cdot 2\right) + \left(4\tfrac{3}{4} \cdot \tfrac{1}{2}\right) \quad \leftarrow \text{Use the Distributive Property}$$

$$= \left(9\tfrac{1}{2}\right) + \left(2\tfrac{3}{8}\right) \quad \leftarrow \text{Multiply.}$$

$$= 11\tfrac{7}{8} \quad \leftarrow \text{Add.}$$

Vocabulary Tip

The Distributive Property shows how multiplication affects addition or subtraction:
$a(b + c) = ab + ac$.

Example

1 **Multiplying Positive Rational Numbers** When both factors are positive, the product is positive

$$3\tfrac{1}{3} \times 1\tfrac{1}{2} = \left(\boxed{3 + \tfrac{1}{3}} \times 1\tfrac{1}{2}\right) \quad \leftarrow \text{Write } 3\tfrac{1}{3} \text{ as a } \boxed{\text{sum}}.$$

$$= \left(\boxed{3} \times 1\tfrac{1}{2}\right) + \left(\boxed{\tfrac{1}{3}} \times 1\tfrac{1}{2}\right) \quad \leftarrow \text{Use the Distributive Property.}$$

$$= \boxed{4\tfrac{1}{2}} + \boxed{\tfrac{1}{2}} \quad \leftarrow \text{Multiply.}$$

$$= 5 \quad \leftarrow \boxed{\text{Add}}.$$

Quick Check

1. Find the product. Write your answer in simplest form.

a. $4.75 \cdot 2.2$ **b.** $\left(8\frac{2}{5}\right)\left(\frac{15}{24}\right)$ **c.** $3.7 \cdot 5.1$

EXAMPLE **Multiplying Negative Rational Numbers**

2 When both factors are negative, the product is positive.

Find $(-3.2)(-4.1)$.

$$(-3.2)(-4.1) = (-1 \cdot 3.2)(-1 \cdot 4.1) \quad \leftarrow \text{Write the negative factors as products.}$$

$$= -1 \cdot (3.2 \cdot -1) \cdot 4.1 \quad \leftarrow \text{Use the Associative Property of Multiplication.}$$

$$= -1 \cdot (-1 \cdot 3.2) \cdot 4.1 \quad \leftarrow \text{Use the Commutative Property of Multiplication.}$$

$$= (-1 \cdot -1) \cdot (3.2 \cdot 4.1) \quad \leftarrow \text{Use the Associative Property of Multiplication.}$$

$$= (1) \cdot (3.2 \cdot 4.1) \quad \leftarrow (-1)(-1) = 1$$

$$= 13.12 \quad \leftarrow \text{Multiply.}$$

Example

2 **Multiplying Negative Rational Numbers** When both factors are negative, the product is positive.

$(-2.2)(-10.4) = (-1 \cdot 2.2)(-1 \cdot 10.4)$ ← Write the negative factors as products.

$= -1 \cdot (2.2 \cdot -1) \cdot 10.4$ ← Use the [redacted] Property of Multiplication

$= -1 \cdot (-1 \cdot 2.2) \cdot 10.4$ ← Use the [redacted] Property of Multiplication

$= (-1 \cdot -1) \cdot (2.2 \cdot 10.4)$ ← Use the [redacted] Property of Multiplication

$= (1)(2.2 \cdot 10.4) =$ [redacted] ← Multiply.

Quick Check

2. Find the product. Write your answer in simplest form.

a. $\left(-2\frac{2}{3}\right)\left(-2\frac{1}{4}\right)$ [redacted]

b. $-7.5 \cdot (-3.1)$ [redacted]

c. $\left(-2\frac{2}{5}\right)\left(-1\frac{1}{3}\right)$ [redacted]

EXAMPLE Multiplying With Different Signs

3 When both factors have different signs, the product is negative.

Find $(-6.3)(2.2)$.

$(-6.3)(2.2) = (-1 \cdot 6.3)(2.2)$ ← Write the negative factor as a product.

$= -1 \cdot (6.3 \cdot 2.2)$ ← Use the Associative Property of Multiplication.

$= -1 \cdot (13.86)$ ← Multiply.

$= -13.86$ ← Simplify.

Example

3 **Multiplying with Different Signs** When both factors have different signs, the product is negative.

Find $(-5.1)(1.5)$.

$(-5.1)(1.5) = (-1 \cdot 5.1)(1.5)$ ← Write the negative factor as a product.

$= (-1)(5.1 \cdot 1.5)$ ← Use the Associative Property of Multiplication.

$= -7.65$ ← Multiply.

Quick Check

3. Find the product. Write your answer in simplest form.

a. $\left(-\frac{3}{5}\right)\left(4\frac{1}{6}\right)$ **b.** $-8.5 \cdot (1.2)$ **c.** $\left(-1\frac{11}{16}\right)\left(\frac{8}{9}\right)$

EXAMPLE **Application: Freediving**

4 Freediving is an underwater activity in which the diver does not use scuba gear. When James freedives, he can descend $1\frac{1}{3}$ ft per second below sea level. How far below sea level can James descend in 5 seconds?

$\left(-1\frac{1}{3}\right)(5) = \left(-\frac{4}{3}\right)\left(\frac{5}{1}\right)$ ← Use multiplication to write an expression for the amount.

$= \left(-\frac{20}{3}\right)$ ← Multiply to find the distance below sea level.

$= -6\frac{2}{3}$ ← Simplify.

James descends to $-6\frac{2}{3}$ ft.

Example

4 Application: Freediving If Sofia freedives and descends $\frac{7}{8}$ ft per second below sea level, how far can she descend in 16 seconds?

$\left(-\frac{7}{8}\right)(16) = \left(-\frac{7}{8}\right)\left(\frac{16}{1}\right)$ ← Use multiplication to write an expression for the amount.

$= [\quad]$ ← Multiply to find the distance below sea level.

Sofia descends [ ]

You have a 5 question worksheet to practice multiplying rational numbers. It is due tomorrow.

Name: _____ Period: _____ September _____2013

Find each product.

1. 2.8 x 0.05 _____

2. 5/8 x 2/5 _____

3. 2 3/5 x 7/8 _____

4. 5 x (-1 2/3) _____

5. -3.8 x (-912) _____