

4-1

Multiplying Fractions and Mixed Numbers

CONTENT STANDARDS
Essential for understanding
6.NS.1

What You'll Learn

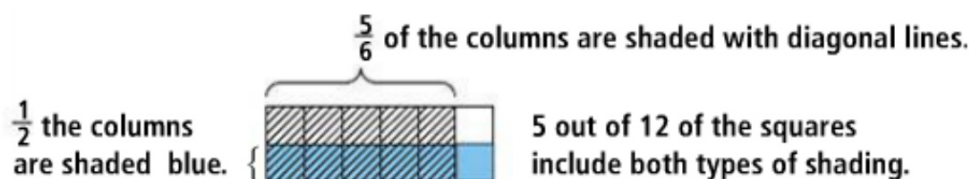
To solve problems by multiplying fractions and multiplying mixed numbers

Why Learn This?

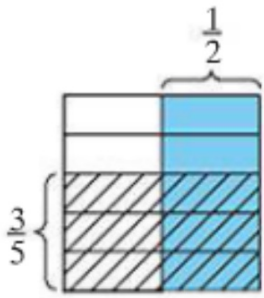
Suppose you are building a model. You want to know the length of half of a $\frac{5}{6}$ -inch piece of wood. You can multiply fractions to find part of a fractional quantity.



The model below shows $\frac{1}{2} \times \frac{5}{6}$.



So $\frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$. You can also find this product by multiplying the numerators and multiplying the denominators.



EXAMPLE

Multiplying Two Fractions

1 Find $\frac{3}{5}$ of $\frac{1}{2}$.

$$\begin{aligned} \frac{3}{5} \cdot \frac{1}{2} &= \frac{3 \cdot 1}{5 \cdot 2} && \leftarrow \text{Multiply the numerators.} \\ & && \leftarrow \text{Multiply the denominators.} \\ &= \frac{3}{10} && \leftarrow \text{Simplify.} \end{aligned}$$

Example

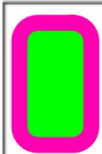
1 **Multiplying Two Fractions** Find $\frac{5}{6}$ of $\frac{3}{8}$.

$$\begin{aligned} \frac{5}{6} \cdot \frac{3}{8} &= \frac{5 \cdot \boxed{3}}{\boxed{6} \cdot 8} && \leftarrow \text{Multiply the numerators.} \\ & && \leftarrow \text{Multiply the denominators.} \\ &= \frac{\boxed{15}}{\boxed{48}} && \leftarrow \text{Find the two products.} \\ &= \frac{\boxed{5}}{\boxed{16}} && \leftarrow \text{Simplify.} \end{aligned}$$

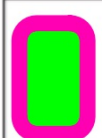
Quick Check

Table talk about these problems

1. a. Find $\frac{3}{5} \cdot \frac{1}{4}$.



b. Find $\frac{2}{9} \times \frac{5}{7}$.



KEY CONCEPTS

Multiplying Fractions

Arithmetic

$$\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$$

Algebra

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \text{ where } b \text{ and } d \text{ are not zero.}$$

When the numerators and the denominators have a common factor, you can simplify before multiplying fractions.

$$\begin{aligned} \frac{3}{8} \cdot \frac{2}{5} &= \frac{3 \cdot \cancel{2}^1}{\cancel{8}_4 \cdot 5} && \leftarrow \text{Divide 8 and 2 by their GCF, 2.} \\ &= \frac{3 \cdot 1}{4 \cdot 5} && \leftarrow \text{Multiply the numerators and the denominators.} \\ &= \frac{3}{20} && \leftarrow \text{Simplify.} \end{aligned}$$

Vocabulary Tip

The word *of* usually suggests multiplication.

To multiply a fraction by a whole number, write the whole number as an improper fraction with a denominator of 1.

EXAMPLE Multiplying a Whole Number



- 2 You are decorating a bulletin board using a piece of green ribbon that is $\frac{5}{6}$ yard long. You also need yellow ribbon that is nine times as long as the green ribbon. How much yellow ribbon do you need?

Draw a picture to help see how these lengths are related.



Find the length of the yellow ribbon by multiplying 9 and $\frac{5}{6}$.

$$\begin{aligned} 9 \cdot \frac{5}{6} &= \frac{9}{1} \cdot \frac{5}{6} && \leftarrow \text{Write 9 as } \frac{9}{1}. \\ &= \frac{\cancel{9}^3}{1} \cdot \frac{5}{\cancel{6}_2} && \leftarrow \text{Divide 9 and 6 by their GCF, 3.} \\ &= \frac{3 \cdot 5}{1 \cdot 2} && \leftarrow \text{Multiply the numerators and denominators.} \\ &= \frac{15}{2}, \text{ or } 7\frac{1}{2} && \leftarrow \text{Simplify. Write as a mixed number.} \end{aligned}$$

The yellow ribbon is $7\frac{1}{2}$ yards long.

Example

② **Multiplying a Whole Number** Find $\frac{2}{5}$ of 30.

$$\frac{2}{5} \cdot 30 = \frac{2}{5} \cdot \frac{\boxed{30}}{\boxed{1}} \quad \leftarrow \text{Write 30 as } \frac{\boxed{30}}{\boxed{1}}.$$

$$= \frac{\boxed{1}}{\boxed{1}} \cdot \frac{2}{\cancel{5}^{\boxed{6}}} \cdot \frac{\cancel{30}_1}{\boxed{1}} \quad \leftarrow \text{Divide 30 and 5 by their GCF, } \boxed{5}.$$

$$= \frac{\boxed{12}}{\boxed{1}} \quad \leftarrow \text{Multiply the numerators and denominators.}$$

$$= \boxed{12} \quad \leftarrow \text{Simplify.}$$

Table talk about this problem

Quick Check

2. A baby alligator is $\frac{5}{6}$ foot long. An adult alligator is 12 times as long as the baby alligator. How long is the adult alligator?



To find the product of mixed numbers, write each mixed number as an improper fraction before multiplying.

EXAMPLE Multiplying Mixed Numbers

3 Find the product $2\frac{2}{3} \times 3\frac{1}{4}$.

Estimate $2\frac{2}{3} \times 3\frac{1}{4} \approx 3 \times 3$, or 9

$$2\frac{2}{3} \times 3\frac{1}{4} = \frac{8}{3} \times \frac{13}{4} \quad \leftarrow \text{Write the mixed numbers as improper fractions.}$$

$$= \frac{\cancel{2}^2}{3} \times \frac{13}{\cancel{4}_1} \quad \leftarrow \text{Divide 8 and 4 by their GCF, 4.}$$

$$= \frac{26}{3}, \text{ or } 8\frac{2}{3} \quad \leftarrow \text{Multiply the numerators and the denominators. Then write the product as a mixed number.}$$

Check for Reasonableness $8\frac{2}{3}$ is near the estimate of 9, so the answer is reasonable.

Example

3 Multiplying Mixed Numbers Find the product $3\frac{3}{8} \times 1\frac{5}{9}$.

Estimate $3\frac{3}{8} \times 1\frac{5}{9} \approx 3 \times 2$, or 6.

$$3\frac{3}{8} \times 1\frac{5}{9} = \frac{27}{8} \times \frac{14}{9} \quad \leftarrow \text{Write the mixed numbers as improper fractions.}$$

$$= \frac{\overset{3}{\cancel{27}}}{\underset{4}{\cancel{8}}} \times \frac{\overset{7}{\cancel{14}}}{\underset{1}{\cancel{9}}} \quad \leftarrow \begin{array}{l} \text{Divide 27 and 9 by their GCF, 3.} \\ \text{Divide 8 and 14 by their GCF, 2.} \end{array}$$

$$= \frac{21}{4}, \text{ or } 5\frac{1}{4} \quad \leftarrow \text{Multiply the numerators and denominators. Then write as a mixed number.}$$

Check for Reasonableness $5\frac{1}{4}$ is near the estimate of 6.

table talk about these problems

Quick Check

3. a. Find $10\frac{1}{4} \times 2\frac{3}{4}$.

b. Find $7\frac{1}{3} \times 3\frac{3}{4}$.

EXAMPLE Application: Skiing

- 4 A student skis $3\frac{1}{2}$ miles in an hour. An instructor can ski $1\frac{1}{3}$ times as far in an hour. How far does the instructor ski in an hour?

The diagram shows the distance that the student skis in one hour. The instructor skis $1\frac{1}{3}$ times as far as the student skis.



$$\text{number of miles the instructor skis} = 1\frac{1}{3} \times \text{number of miles the student skis}$$

$$= 1\frac{1}{3} \times 3\frac{1}{2}$$

$$= \frac{4}{3} \times \frac{7}{2}$$

$$= \frac{2\cancel{4}}{3} \times \frac{7}{\cancel{2}_1}$$

$$= \frac{14}{3}, \text{ or } 4\frac{2}{3}$$

← Write the mixed numbers as improper fractions.

← Divide 4 and 2 by their GCF, 2.

← Multiply the numerators and the denominators. Then write the product as a mixed number.

The instructor skis $4\frac{2}{3}$ miles in one hour.



Check Your Understanding

1. **Number Sense** If you multiply 6 by $\frac{1}{2}$, is the answer greater than or less than the result of multiplying 6 by $\frac{1}{3}$? Explain.

Change each mixed number to an improper fraction.

2. $1\frac{2}{5}$

3. $3\frac{1}{3}$

4. $2\frac{2}{3}$

Find each product.

5. $\frac{2}{3} \cdot \frac{3}{4}$

6. $2\frac{1}{2} \times 1\frac{2}{3}$

7. $2\frac{1}{2} \cdot 1\frac{1}{4}$

You have an assignment worksheet and time to begin working on it now.

Practice 4-1 Multiplying Fractions and Mixed Numbers

find each product.

1. $\frac{1}{6} \times \frac{3}{4}$

2. $\frac{2}{3} \times \frac{1}{2}$

Find each product.

3. $\frac{2}{5}$ of 10

4. $\frac{1}{4}$ of 12

5. $\frac{2}{3}$ of 6

6. $\frac{1}{2} \times \frac{5}{6}$

7. $\frac{3}{4} \times \frac{7}{8}$

8. $\frac{2}{5} \times \frac{7}{11}$

9. $2\frac{5}{8} \cdot 1\frac{3}{4}$

10. $3\frac{3}{8} \cdot 7\frac{1}{4}$

11. $5\frac{3}{8} \times 2\frac{7}{8}$

12. $2\frac{3}{8} \cdot 4\frac{4}{5}$

13. $6\frac{7}{12} \times 5\frac{9}{10}$

14. $7\frac{1}{3} \times 10\frac{11}{12}$

15. $12\frac{1}{4} \times 3\frac{3}{4}$

16. $8\frac{1}{6} \cdot 2\frac{1}{4}$

17. $15\frac{2}{3} \cdot 5\frac{5}{7}$

18. What product does the model represent?



Solve.

19. A kitten eats $\frac{1}{4}$ cup of cat food. Another cat in the same household eats 6 times as much. How much food does the cat eat?

20. Ken used a piece of lumber to build a bookshelf. If he made three shelves that are each $2\frac{1}{2}$ ft long, how long was the piece of lumber?

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