Today, we are working with Irrational Numbers and Square Roots. We introduced this topic last year, and will build on our knowledge today.

We will be able to classify numbers as rational or irrational, and determine what numbers are perfect squares.

Taking notes, especially over the definitions of the new vocabulary terms would be a good idea.

Your assignment is an online math gizmo.

Why Learn This?

Not every situation can be modeled using the four basic operations. For example, you need square roots to relate the time and distance a skydiver falls.

A number that is the square of a whole number is a **perfect square.** The **square root** of a number is another number that when multiplied by itself is equal to the given number.

In the diagram at the right, 16 square tiles form a square with 4 tiles on each side. Since $4 \cdot 4 = 16$ and $-4 \cdot (-4) = 16$, 16 has two square roots, 4 and -4. Since $4^2 = 16$, 16 is a perfect square.

$4^2 = 16$

Perfect Squares

n	n^2
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144

EXAMPLE 1 Finding Square Roots of Perfect Squares

Find the two square roots of 25.

$5 \cdot 5 = 25$ and $-5 \cdot (-5) = 25$

The square roots of 25 are 5 and -5.

Quick Check

1. Find the square roots of each number.

a. 36 [redacted] **b.** 1 [redacted] **c.** $\frac{1}{16}$ [redacted]

1 EXAMPLE Find the two square roots of 81.

$9 \cdot 9 = 81$

$-9 \cdot (-9) = 81$

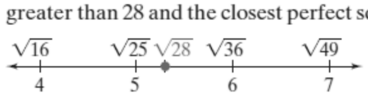
The two square roots of 81 are 9 and –9.

The symbol $\sqrt{\ }$ means the square root of a number. In this book, $\sqrt{\ }$ means the nonnegative square root, unless stated otherwise. So $\sqrt{9}$ means the nonnegative square root of 9, or 3, and $-\sqrt{9}$ means the opposite of the nonnegative square root of 9, or -3.

EXAMPLE Estimating a Square Root

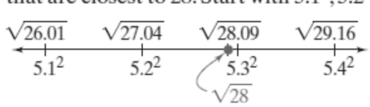
2 Estimate the value of $\sqrt{28}$ to the nearest integer and to the nearest tenth.

To estimate $\sqrt{28}$ to the nearest integer, find the closest perfect square greater than 28 and the closest perfect square less than 28.



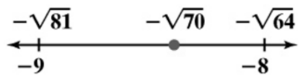
The perfect squares closest to 28 are 25 and 36. Since 28 is closer to 25 than it is to 36, $\sqrt{28}$ must be closer to 5 than to 6. So $\sqrt{28} \approx 5$.

To estimate $\sqrt{28}$ to the nearest tenth, find two squares between 5 and 6 that are closest to 28. Start with $5.1^2, 5.2^2$, and so on.



The squares closest to 28 are 27.04 and 28.09. Since 28 is closer to 28.09 than it is to 27.04, $\sqrt{28}$ must be closer to 5.3 than to 5.2. So, $\sqrt{28} \approx 5.3$.

2 EXAMPLE Estimate the value of $-\sqrt{70}$ to the nearest integer.



Since 70 is closer to 64 than it is to 81, $-\sqrt{70} \approx -8$.

You can compare square roots in the same way you compare integers.

EXAMPLE Comparing Square Roots

3 Which is greater, $\sqrt{18}$ or 4.4?

First, estimate the value of the square root to the nearest tenth.

$\sqrt{18} \approx 4.2$

Since $4.2 < 4.4$, $\sqrt{18} < 4.4$.

Quick Check

3. Which is greater $\sqrt{20}$ or 4.7?

3 EXAMPLE The math class drops a small ball from the top of a stairwell. They measure the distance to the basement as 48 feet. Use the formula $d = 16t^2$ to find how long it takes the ball to fall.

$d = 16t^2$ ⟵ Use the formula.

$48 = 16t^2$ ⟵ Substitute 48 for d.

$\frac{48}{16} = t^2$ ⟵ Divide each side by 16.

$3 = t^2$ ⟵ Simplify.

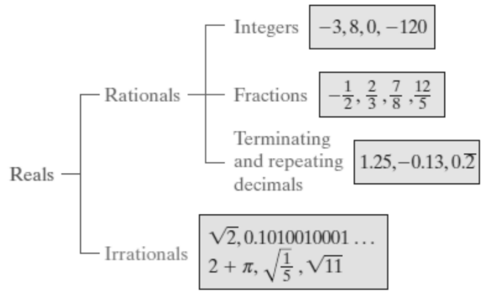
$\sqrt{3} = t$ ⟵ Find the positive square root.

[√] 3 [ENTER] 1.7320508 ⟵ Use a calculator.

$1.7 \approx t$ ⟵ Round to the nearest tenth.

It takes about 1.7 seconds for the ball to fall 48 ft.

Irrational numbers are numbers that cannot be written in the form $\frac{a}{b}$, where a is any integer and b is any nonzero integer. Rational and irrational numbers form the set of **real numbers.** The diagram below shows the relationships among sets of numbers.



The decimal expansion of irrational numbers do not terminate or repeat. The decimal digits of $\pi = 3.14159265359\ldots$ do not terminate or repeat, so π is an irrational number. Irrational numbers can also include decimals that have a pattern in their digits, like 0.02022022202222 . . .

For any integer n that is not a perfect square, $\sqrt{n}$ is irrational.

EXAMPLE Classifying Real Numbers

5 Is each number *rational* or *irrational*? Explain.

a. 0.818118111 . . . Irrational; the decimal does not terminate or repeat.

b. $-0.\overline{81}$ Rational; the decimal repeats.

c. $1\frac{2}{9}$ Rational; the number can be written as the ratio $\frac{11}{9}$.

d. $\sqrt{2}$ Irrational; 2 is not a perfect square.

Quick Check

5. Is $0.\overline{6}$ *rational* or *irrational*? Explain.

4 EXAMPLE Identify each number as *rational* or *irrational*. Explain.

a. $-9.333\overline{3}$

b. $4\frac{7}{9}$

c. $\sqrt{90}$

d. 6.36366366636666. . .

Your assignment is a math gizmo
Log onto your computer.
Use Internet Explorer.
Log into www.pearsonsuccessnet.com
with your username and password.
Go to the first "student resources"
Wait for further instructions.