

Exploring Two-Step Equations

What You'll Learn

To write and evaluate expressions with two operations and to solve two-step equations using number sense

Why Learn This?

Suppose you are ordering roses online. Roses cost \$5 each, and shipping costs \$10. Your total cost depends on how many roses you buy. Two-step equations can help you solve everyday problems.



You can write expressions with variables using one operation. Now you will write algebraic expressions with two operations.

EXAMPLES

Writing and Evaluating Expressions

- 1** Define a variable and write an algebraic expression for the phrase “\$10 plus \$5 times the number of roses ordered.”

Let n = the number of roses ordered ← Define the variable.

$10 + 5 \cdot n$ ← Write an algebraic expression.

$10 + 5n$ ← Rewrite $5 \cdot n$ as $5n$.

- 2** Evaluate the expression for 12 roses.

$10 + 5n$


$10 + 5 \cdot 12$ ← Evaluate the expression for 12 roses.

$10 + 60$ ← Multiply.

70 ← Simplify.

If you order 12 roses, you will have to pay \$70.

1 EXAMPLE To rent a bicycle, you pay a \$12 basic fee plus \$2 per hour. Write an expression for the total cost in dollars of a bicycle rental.

Words	cost per hour	times	number of hours	plus	basic fee
	Let h	=	number of hours rented.		
Expression	2	•	h	+	12

An expression is $2h + 12$

Evaluate the expression for an 8-hour bicycle rental.

$$2 \cdot 8 + 12 \quad \leftarrow \text{Evaluate the expression for 8 hours.}$$

$$= 16 + 12 \quad \leftarrow \text{Multiply.}$$

$$= 28 \quad \leftarrow \text{Add.}$$

For an 8-hour bicycle rental and the basic fee, the total cost is \$28.

Examples

- ① **Writing Expressions** Define a variable and write an algebraic expression for the phrase “four times the length of a rope in inches, increased by eight inches.”

Let \boxed{l} = length of rope in inches. ← Define the variable.

$\boxed{4} \cdot \boxed{l} + \boxed{8}$ ← Write an algebraic expression.

$\boxed{4l} + 8$ ← Rewrite $4 \cdot \ell$ as $\boxed{4l}$.

2 EXAMPLE Solve $9k - 4 = 14$ using number sense.

$$9k - 4 = 14$$

$$\square - 4 = 14$$

← Cover $9k$. *Think:* What number minus 4 is 14? Answer: 18

$$9k = 18$$

← So \square , or $9k$, must equal 18.

$$9 \cdot \square = 18$$

← *Think:* What number times 9 is 18? Answer: 2

$$k = 2$$

← So \square , or k , must equal 2.

- 2 Evaluating Expressions** Evaluate the expression if the length of a rope is 9 inches.

$$4\ell + 8$$

$4 \cdot \boxed{} + 8$ ← Evaluate the expression for a rope length of $\boxed{}$.

$\boxed{} + 8$ ← Multiply.

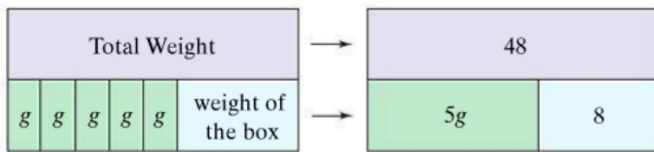
$\boxed{}$ ← Simplify.

Suppose your grandmother sends you 5 games for your birthday. Each game has the same weight. The box she mails them in weighs 8 ounces. The total weight is 48 ounces. What is the weight of one game?



You can represent this situation with the diagram below.

Let g represent the weight of a game.



You can solve this problem using the equation $5g + 8 = 48$. Since there is more than one operation in the equation, there will be more than one step in the solution.

EXAMPLE**Using Number Sense**

3 Solve $5g + 8 = 48$ by using number sense.

$$5g + 8 = 48$$

$$\square + 8 = 48 \quad \leftarrow \text{Cover } 5g. \text{ Think: What number added to 8 is 48? Answer: 40}$$

$$5g = 40 \quad \leftarrow \text{So } \square, \text{ or } 5g, \text{ must equal 40.}$$

$$5 \cdot \square = 40 \quad \leftarrow \text{Now cover } g. \text{ Think: What number times 5 is 40? Answer: 8}$$

$$g = 8 \quad \leftarrow \text{So } \square, \text{ or } g, \text{ must equal 8.}$$

Check

$$5g + 8 = 48 \quad \leftarrow \text{Check your solution in the original equation.}$$

$$5(8) + 8 \stackrel{?}{=} 48 \quad \leftarrow \text{Substitute 8 for } g.$$

$$40 + 8 \stackrel{?}{=} 48 \quad \leftarrow \text{Simplify.}$$

$$48 = 48 \quad \checkmark \quad \leftarrow \text{The solution checks.}$$

3 EXAMPLE The Healy family wants to buy a DVD player that costs \$200. They already have \$80 saved toward the cost. How much will they have to save per month for the next six months in order to have the whole cost saved?

Words amount already saved plus (amount to save per month • 6) is \$200



Let z = amount to save per month.

Equation 80 + $(z \cdot 6)$ = 200

$$80 + 6z = 200$$

$$80 + \square = 200 \quad \leftarrow \text{Cover } 6z. \text{ Think: What number added to 80 is 200? Answer: 120.}$$

$$6z = 120 \quad \leftarrow \text{So } \square, \text{ or } 6z, \text{ must equal 120.}$$

$$6 \square = 120 \quad \leftarrow \text{Think: What number times 6 is 120? Answer: 20.}$$

$$z = 20 \quad \leftarrow \text{So } \square, \text{ or } z, \text{ must equal 20.}$$

They will have to save \$20 per month.

3 Using Number Sense Solve $3n - 4 = 14$ by using number sense.

$$3n - 4 = 14$$

$$\blacksquare - 4 = 14$$

← Cover $3n$. *Think:* What number minus 4 is 14?

Answer: .

$$3n = \text{18}$$

← So \blacksquare , or $3n$, must equal .

$$3 \cdot \blacksquare = \text{18}$$

← Now cover n . *Think:* What number times 3 is .

Answer: .

$$n = \text{6}$$

← So \blacksquare , or n , must equal .

Check

$$3n - 4 = 14$$

← Check your solution in the original equation.

$$3(6) - 4 \stackrel{?}{=} 14$$

← Substitute 6 for n .

$$18 - 4 \stackrel{?}{=} 14$$

← Simplify.

$$14 = 14$$

← The solution checks.

Test Prep Tip

You can represent the relationships in the problem with this model.

share of total	
lemonade cost	share of chicken cost
\$5.50	
\$1.50	$\frac{z}{3}$

EXAMPLE Application: Food

- 4 Suppose you buy a jumbo lemonade for \$1.50 and divide the cost of an order of chicken wings with two friends. Your share of the total bill is \$5.50. Write and solve an equation to find the cost of the chicken wings.

Words cost of lemonade plus (cost of wings \div 3) is \$5.50

Let z = the cost of the chicken wings.

Expression 1.50 + $(z \div 3)$ = \$5.50

$$1.50 + \frac{z}{3} = 5.50$$

$$1.50 + \square = 5.50 \quad \leftarrow \text{Cover } \frac{z}{3}. \text{ Think: What number added to } 1.50 \text{ is } 5.50? \text{ Answer: } 4$$

$$\frac{z}{3} = 4 \quad \leftarrow \text{So } \square, \text{ or } \frac{z}{3}, \text{ must equal } 4.$$

$$\frac{\square}{3} = 4 \quad \leftarrow \text{Now cover } z. \text{ Think: What number divided by } 3 \text{ is } 4? \text{ Answer: } 12$$

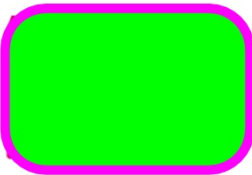
$$z = 12 \quad \leftarrow \text{So } \square, \text{ or } z, \text{ must equal } 12.$$

The cost of the chicken wings is \$12.



Check Your Understanding

1.



1. **Vocabulary** What is the difference between a one-step expression and a two-step expression? **See below left.**

2. Write and solve the equation modeled at the left.



Match each phrase with the correct algebraic expression.

3. 10 centimeters less than twice x , your hand length A. $\frac{1}{2}x - 10$

4. 10 people fewer than half x , the town's population B. $2x - 10$

5. 10 more than two times a number x C. $2x + 10$

Using number sense, fill in the missing number.

6. $4b + 5 = 17$

$4b = \blacksquare$

7. $7c - 20 = 50$

$7c = \blacksquare$

You have an assignment worksheet and time to begin working now.

Reteaching 2-4

Exploring Two-Step Equations

You can change a word expression into an algebraic expression by converting the words to variables, numbers, and operation symbols.

To write a two-step algebraic expression for *seven more than three times a number*, follow these steps.

- | | |
|--|--|
| ① Define the variable. | Let n represent the number. |
| ② Ask yourself if there are any key words. | “More than” means add and
“times” means multiply. |
| ③ Write an algebraic expression. | $7 + 3 \cdot n$ |
| ④ Simplify. | $7 + 3n$ |

Define a variable and write an algebraic expression for each phrase.

- 3 inches more than 4 times your height _____
- 4 less than 6 times the weight of a turkey _____
- 8 more than one-half the number of miles run last week _____

Solve.

- Three friends pay \$4 per hour to rent a paddleboat plus \$5 for snacks. Write an expression for the total cost of rental and snacks. Then evaluate the expression for 2 hours.

- A lawn care service charges \$10 plus \$15 per hour to mow and fertilize lawns. Write an expression for the total cost of having your lawn mowed and fertilized. Then evaluate the expression for 4 hours.

Solve each equation using number sense.

- $4x - 10 = 30$ _____
- $2n - 7 = 13$ _____
- $\frac{x}{3} + 2 = 4$ _____