

## 7-6


# Angles and Polygons

### CONTENT STANDARDS

8.G.5

### What You'll Learn

To find the angle measures of a polygon

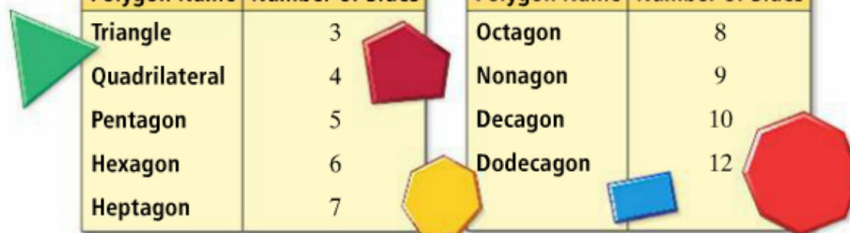
 **New Vocabulary** interior angle, exterior angle

### Why Learn This?

Polygons often appear in art and architecture. In designing tile patterns, it helps to know about the angles of polygons.

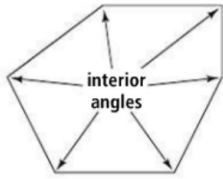
Here is a list of common polygons.

Polygon Name	Number of Sides	Polygon Name	Number of Sides
Triangle	3	Octagon	8
Quadrilateral	4	Nonagon	9
Pentagon	5	Decagon	10
Hexagon	6	Dodecagon	12
Heptagon	7		



*I recommend putting the names of the polygons and the number of their sides into your notes.*

**Please put this page in your notes**



Two consecutive sides of a polygon form one **interior angle**. The sum of the measures of the interior angles depends on the number of sides.

**KEY CONCEPTS** Polygon Angle Sum

For a polygon with  $n$  sides, the sum of the measures of the interior angles is  $(n - 2)180^\circ$ .

**EXAMPLE**

**Sum of the Interior Angle Measures**

**1** What is the sum of the measures of the interior angles of a nonagon?

$$\begin{aligned}(n - 2)180^\circ &= (9 - 2)180^\circ \quad \leftarrow \text{A nonagon has nine sides. Substitute 9 for } n. \\ &= 1,260^\circ \quad \leftarrow \text{Simplify.}\end{aligned}$$

The sum of the interior angle measures of a nonagon is  $1,260^\circ$ .

### Example

- ① **Sum of the Interior Angle Measures** Find the sum of the measures of the interior angles of an octagon.

An octagon has 8 sides.

$$(n - 2)180^\circ = (\boxed{8} - 2)180^\circ \leftarrow \text{Substitute 8 for } n.$$
$$= \boxed{1080}^\circ \leftarrow \text{Simplify.}$$

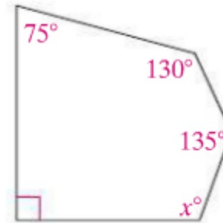
### Quick Check

1. What is the sum of the measures of the interior angles of a heptagon?

You can use the same formula to find angle measures in a polygon.

## EXAMPLE Angle Measures of a Polygon

- 2 **Algebra** Find the unknown angle measure in the pentagon at the right.



**Step 1** Find the sum of the angle measures.

$$\begin{aligned}(n - 2)180^\circ &= (5 - 2)180^\circ && \leftarrow \text{Substitute 5 for } n. \\ &= 540^\circ && \leftarrow \text{Simplify.}\end{aligned}$$

**Step 2** Write an equation. Let  $x$  = the unknown angle measure.

$$540^\circ = 90^\circ + 75^\circ + 130^\circ + 135^\circ + x^\circ \quad \leftarrow \text{Write an equation.}$$

$$540^\circ = 430^\circ + x^\circ \quad \leftarrow \text{Simplify.}$$

$$110^\circ = x^\circ \quad \leftarrow \text{Subtract } 430^\circ \text{ from each side.}$$

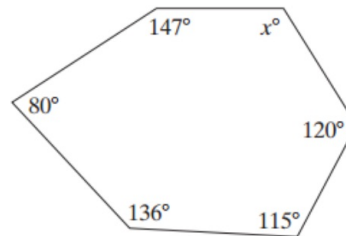
The unknown angle measure is  $110^\circ$ .

### Examples

- 2 **Angle Measures of a Polygon** Find the missing angle measure in the hexagon.

**Step 1** Find the sum of the measures of the interior angles of a hexagon.

$$\begin{aligned}(n - 2)180^\circ &= (\boxed{6} - 2)180^\circ && \leftarrow \text{Substitute 6 for } n. \\ &= \boxed{720} && \leftarrow \text{Simplify.}\end{aligned}$$



**Step 2** Write an equation.

Let  $x$  = the missing angle measure.

$$\boxed{720} = 120^\circ + \boxed{147} + 136^\circ + \boxed{80} + \boxed{115} + x^\circ$$

← Write an equation.

$$720^\circ = \boxed{598} + x^\circ$$

← Add.

$$\boxed{122} = x^\circ$$

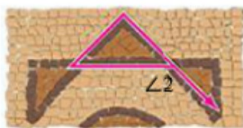
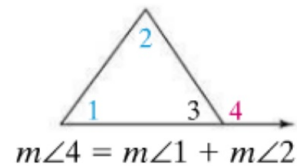
← Subtract  $598^\circ$  from each side.

The missing angle measure is  $\boxed{122}$ .

Consider taking a picture of this page for your notes.

An **exterior angle** of a polygon is an angle formed by a side and an extension of an adjacent side.

The measure of an exterior angle of a triangle is equal to the sum of the measures of the interior angles at the other two vertices.



An ancient Roman mosaic made of tiles

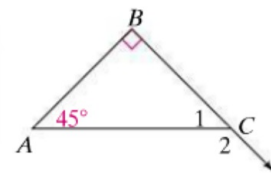
### EXAMPLE

### Finding the Measure of an Exterior Angle

- 3 **Art** The diagram shows a portion of the design of a mosaic.  $\angle 2$  is an exterior angle of  $\triangle ABC$ . What is  $m\angle 2$ ?

$$\begin{aligned} m\angle 2 &= m\angle A + m\angle B && \leftarrow \text{Exterior angle of triangle} \\ &= 45^\circ + 90^\circ && \leftarrow \text{Substitute.} \\ &= 135^\circ && \leftarrow \text{Simplify.} \end{aligned}$$

$\angle 2$  measures  $135^\circ$ .



**Check** By the angle sum of a triangle,  $m\angle 1 = 180^\circ - 45^\circ - 90^\circ = 45^\circ$ .  
 $\angle 1$  and  $\angle 2$  are supplementary, so  $m\angle 2 = 180^\circ - 45^\circ = 135^\circ$ . ✓

- 3 Finding the Measure of an Exterior Angle**  $\angle 2$  is an exterior angle of  $\triangle ABC$ .  
What is  $m\angle 2$ ?

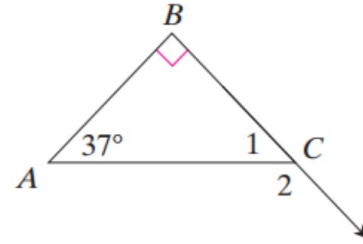
$$\begin{aligned} m\angle 2 &= m\angle A + m\angle B && \leftarrow \text{Exterior angle of triangle} \\ &= \boxed{37} + \boxed{90} && \leftarrow \text{Substitute.} \\ &= \boxed{127} && \leftarrow \text{Simplify.} \end{aligned}$$

$\angle 2$  measures  $127^\circ$ .

**Check**

By the angle sum of a triangle,  $m\angle 1 = 180^\circ - 90^\circ - 37^\circ = 53^\circ$ .

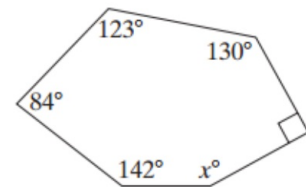
$\angle 1$  and  $\angle 2$  are supplementary, so  $m\angle 2 = 180^\circ - 53^\circ = 127^\circ$ . ✓



**Quick Check**

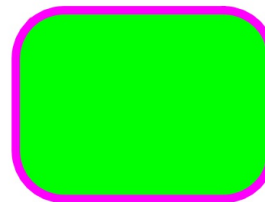
2. A hexagon has five angles with measures of  $142^\circ$ ,  $84^\circ$ ,  $123^\circ$ ,  $130^\circ$ , and  $90^\circ$ . What is the measure of the sixth angle?

3. In  $\triangle RST$ ,  $m\angle R = 63^\circ$  and  $m\angle S = 84^\circ$ . What is the measure of the exterior angle at vertex  $T$ ?

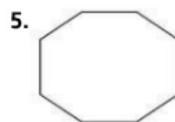
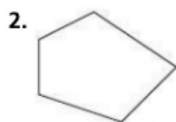


 **Check Your Understanding**

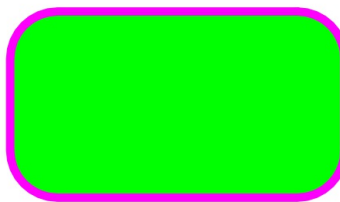
1. **Vocabulary** How does an interior angle of a polygon differ from an exterior angle?



**Classify each polygon by the number of its sides.**



6. **Error Analysis** Jason knows the sum of the angle measures of a hexagon is  $720^\circ$ . To find the sum of the angle measures of a dodecagon, he multiplies  $720^\circ$  by 2 since  $12 = 6 \cdot 2$ . Miranda multiplies  $180^\circ$  by 10. Who is correct? Explain.



**Go to [m.socrative.com](https://www.m.socrative.com)  
room number 262013**

**Practice 7-6** Angles and Polygons

Classify each polygon by its number of sides.

1.



\_\_\_\_\_

2.



\_\_\_\_\_

3.



\_\_\_\_\_

4. a polygon with 8 sides

\_\_\_\_\_

5. a polygon with 10 sides

\_\_\_\_\_

6. Find the measure of each angle of a regular hexagon.

\_\_\_\_\_

7. The measures of four angles of a pentagon are  $143^\circ$ ,  $118^\circ$ ,  $56^\circ$ , and  $97^\circ$ . Find the measure of the missing angle.

\_\_\_\_\_

8. Complete the chart for the total of the angle measures in each polygon. The first three have been done for you.

Polygon	Number of Sides	Sum of Angle Measures
triangle	3	$180^\circ$
rectangle	4	$360^\circ$
pentagon	5	$540^\circ$
hexagon		
heptagon		
octagon		

9. From the table you completed in Exercise 8, what pattern do you see? Explain.

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